Transition to College Math

Name ______________________________

Date ____________ Period _________

<table>
<tr>
<td>Unit: **4: Trigonometric Graphs and Identities**

Lesson : **5: Double-Angle and Half Angle Identities**</td>
<td>Essential Question: **What is the <u>exact</u> value of $\sin 22.5°$? Show your work.**</td>
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<td>Standard: F-TF.9</td>
<td>Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.</td>
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<td>***Learning Target*:**</td>
<td>80% of the students will be able to calculate the horizontal distance a golf ball will travel before hitting the ground if its initial velocity is $v_0 = 50\, m/s$, and its initial trajectory is 30° above the horizontal.

When Los Angeles Dodger Joc Pederson hit a home run in a game against the San Francisco Giants, the ball left the bat with a speed of 101.4 mi/h and a launch angle of 32.3°.

What horizontal distance would the ball have traveled if it had not encountered any obstructions? (Photo courtesy ESPN.) We will use a double angle identity to find the answer.</td>
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<td colspan="2">Summary</td>
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Double-Angle Identities:

$\sin(2\theta) = \sin(\theta + \theta)$

$= \sin\theta\cos\theta + \cos\theta\sin\theta$

$= 2\sin\theta\cos\theta$

The rest of the double-angle identities can be derived similarly. The table below summarizes them.

Double-Angle Identities
$\sin(2\theta) = 2\sin\theta\cos\theta$
$\cos(2\theta) = \cos^2\theta - \sin^2\theta$ $= 2\cos^2\theta - 1$ $= 1 - 2\sin^2\theta$
$\tan(2\theta) = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

Example 1:

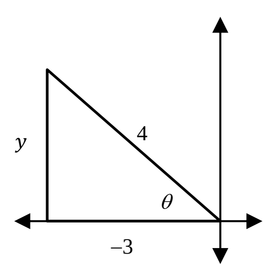
Evaluating expressions using Double-Angle expressions.

Find $\sin 2\theta$ and $\cos 2\theta$ if $\cos\theta = -\frac{3}{4}$ and $90° < \theta < 180°$.

1. Find $\sin\theta$.

   I. Use the reference angle.

   In Quadrant II, this figure illustrates the reference angle θ when $\cos\theta = -\frac{3}{4}$.



   Using the Pythagorean Theorem,

   $(-3)^2 + y^2 = 4^2$

   $y = \sqrt{16 - 9} = \sqrt{7}$

	$\sin\theta = \frac{\sqrt{7}}{4}$ II. Solve $\sin^2\theta = 1 - \cos^2\theta$. $\sin\theta = \sqrt{1 - \left(-\frac{3}{4}\right)^2}$ $= \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4}$ $\sin\theta = \frac{\sqrt{7}}{4}$ 2. Find $\sin 2\theta$. $\sin 2\theta = 2\sin\theta\cos\theta$ $= 2 \cdot \frac{\sqrt{7}}{4}\left(-\frac{3}{4}\right)$ $= -\frac{3\sqrt{7}}{8}$ 3. Find $\cos 2\theta$. $\cos 2\theta = 2\cos^2\theta - 1$ $= 2\left(-\frac{3}{4}\right)^2 - 1$ $= 2\frac{9}{16} - 1 = \frac{1}{8}$

Exercise 1:	Find $\tan 2\theta$ and $\cos 2\theta$ if $\cos\theta = \frac{1}{3}$ and $270° < \theta < 360°$.

***Example 2*:**	Prove: A. $\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$ $\sin^2\theta = 1-\cos^2\theta$ $= 1-\dfrac{1}{2}(\cos 2\theta + 1)$ $= \dfrac{1}{2}-\dfrac{1}{2}\cos 2\theta$ $= \dfrac{1}{2}(1-\cos 2\theta)$ B. $(\cos\theta + \sin\theta)^2 = 1+\sin 2\theta$ $(\cos\theta+\sin\theta)^2 = \cos^2\theta + 2\cos\theta\sin\theta + \sin^2\theta$ $= 1 + 2\sin\theta\cos\theta$ $= 1+\sin 2\theta$
***Exercise 2*:**	Prove: A. $\cos^4\theta - \sin^4\theta = \cos 2\theta$

Exercise 2, continued	B. $$\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$$
Half-Angle Identities:	To find the identity for $\cos\frac{\theta}{2}$, substitute $\frac{\theta}{2}$ for θ in the double angle identity. $$\cos\theta = \cos 2\left(\frac{\theta}{2}\right) = 2\cos^2\left(\frac{\theta}{2}\right) - 1$$ $$2\cos^2\left(\frac{\theta}{2}\right) = 1 + \cos\theta$$ $$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$$ The other half-angle identities can be found in similar ways.

	Half-Angle Identities $\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$ $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$ $\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$ Choose + or – depending on the location of $\frac{\theta}{2}$.

Class work: p 788: 1-12	
Homework: p 788: 14-48 even, 49, 50, 59, 62	