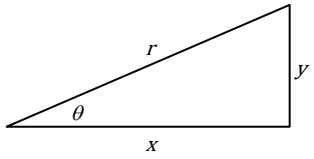
Transition to College Math

Name _____

Date _____ Period _____

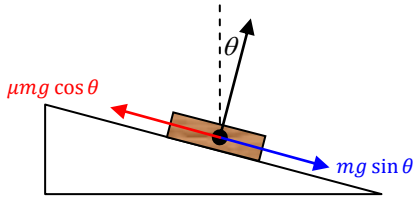
Unit 4: Trigonometric Graphs and Identities **Lesson 3: Fundamental Trigonometric Identities**	Essential Question: **If the coefficient of friction between a worker's shoes and the roof of a building is $\mu = 0.4$, will the worker slide off a roof with a 6:12 pitch? Show your work, and explain your answer.**
Standard: F-TF.8	Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.
***Objective*:**	Use fundamental trigonometric identities to simplify and rewrite expressions and to verify other identities. 80% of the students will be able to prove $\sec^2\theta\,(1-\cos^2\theta) = \tan^2\theta$
Trigonometric Identities:	An identity is a mathematical statement that is true for all values of the variable for which the statement is defined. The following equation is an example of a mathematical for all real values of x and y: $(x+y)^2 = x^2 + 2xy + y^2$ A trigonometric identity is an identity that involves trigonometric functions, such as $\tan x$. Often, we can use trigonometric identities to simplify trigonometric expressions.
Pythagorean Identity:	We can use the figure on the right to derive the Pythagorean identity $x^2 + y^2 = r^2$ Pythagorean Theorem  $\frac{x^2}{r^2} + \frac{y^2}{r^2} = 1$ $\sin^2\theta + \cos^2\theta = 1$ definition of sine and cosine functions.
Summary	

Fundamental Trigonometric Identities:

***Reciprocal*:**	$\sec\theta = \frac{1}{\cos\theta}$ $\csc\theta = \frac{1}{\sin\theta}$ $\cot\theta = \frac{1}{\tan\theta}$
***Tangent and Cotangent Ratios*:**	$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$
***Pythagorean*:**	$\sin^2\theta + \cos^2\theta = 1$ $1 + \tan^2\theta = \sec^2\theta$ $1 + \cot^2\theta = \csc^2\theta$
Negative Angles:	$\sin(-\theta) = -\sin\theta$ $\cos(-\theta) = \cos\theta$ $\tan(-\theta) = -\tan\theta$

***Example 1*:**	Prove 1. $\sec\theta = \csc\theta\tan\theta$ $= \left(\frac{1}{\sin\theta}\right)\left(\frac{\sin\theta}{\cos\theta}\right)$ $= \frac{1}{\cos\theta}$ $= \sec\theta$ 2. $\csc(-\theta) = -\csc\theta$ $= \frac{1}{\sin(-\theta)}$ $= \frac{1}{-\sin\theta}$ $= -\frac{1}{\sin\theta}$ $= -\csc\theta$
***Exercise 1*:**	Prove 1. $\sin\theta\cot\theta = \cos\theta$ 2. $1 - \sec(-\theta) = 1 - \sec\theta$

Example 2*: *Rewriting Trigonometric Expressions	Rewrite each expression in terms of $\cos\theta$, and simplify. 1. $\frac{\sin^2\theta}{1-\cos\theta}$ $= \frac{1-\cos^2\theta}{1-\cos\theta}$ $= \frac{(1+\cos\theta)(1-\cos\theta)}{1-\cos^2\theta}$ $= 1+\cos\theta$ 2. $\sec\theta - \tan\theta\sin\theta$ $= \frac{1}{\cos\theta} - \left(\frac{\sin\theta}{\cos\theta}\right)\sin\theta$ $= \frac{1}{\cos\theta} - \frac{\sin^2\theta}{\cos\theta}$ $= \frac{1-\sin^2\theta}{\cos\theta}$ $= \frac{\cos^2\theta}{\cos\theta}$ $= \cos\theta$
***Exercise 2*:**	Rewrite each expression in terms of $\sin\theta$, and simplify. 1. $\frac{\cos^2\theta}{1-\sin\theta}$ 2. $\cot^2\theta$

***Example 3*:**	A ski company is testing the friction of a new ski wax by placing a waxed wood block on an inclined plane of wet snow. The incline plane is slowly raised until the block begins to slide.  The coefficient μ is the coefficient of static friction, m is the mass of the block, and g is the acceleration of gravity. At the moment the block begins to slide, the two forces are balanced. $$mg\sin\theta = \mu mg\cos\theta$$ If $\mu = 0.14$, what is the angle when the block begins to slide? $$\frac{\sin\theta}{\cos\theta} = \frac{\mu mg}{mg}$$ $$\tan\theta = 0.14$$ $$\theta \approx 8.0°$$
***Exercise 3*:**	If $\mu = 0.44$, what is the angle when the block begins to slide?
Class work:	p 775: 1-7
Homework:	p 775: 8-16, 17-43 odd, 52-55, 62-69