Transition to College Math

Name ______________________________

Period __________

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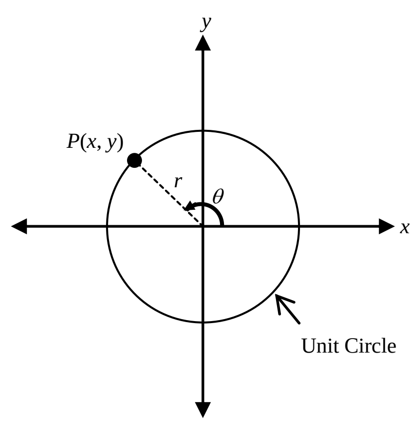
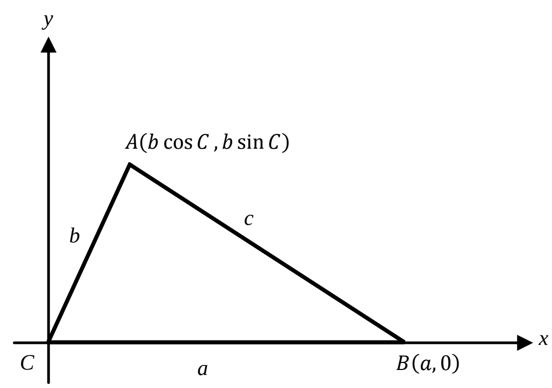
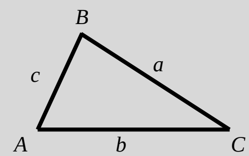
Date: Unit **3: Trigonometry** Lesson **6: The Law of Cosines**	Essential Question: **Explain how to use the Law of Cosines to prove the Pythagorean Theorem.**
Standard: G-SRT.10 G-SRT.11	Prove the Laws of Sines and Cosines and use them to solve problems. Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
***Learning Target*:**	Use the Law of Cosines to find the side measures and angles of a triangle. Use Heron's formula to find the area of a triangle. A triangle has one side of length 5 cm and another side of length 3 cm. The angle between them is 120°. How long is the third side? Consider the unit circle and a point on that circle. 
Summary	

	From a previous lesson, we know the following: $r = 1$ $x = \cos\theta$ $y = \sin\theta$ We also know that $x^2 + y^2 = r^2$ Combining these, we conclude, $\cos^2\theta + \sin^2\theta = 1$
***Proof of the Law of Cosines*:**	Draw ΔABC with $\angle C$ in standard position.  We find the length of c by using the distance formula: $c^2 = (x_A - x_B)^2 + (y_A - y_B)^2$ $= (b\cos C - a)^2 + (b\sin C - 0)^2$ $= b^2\cos^2 C - 2ab\cos C + a^2 + b^2\sin^2 C$ $= a^2 + b^2(\cos^2 C + \sin^2 C) - 2ab\cos C$ $= a^2 + b^2 - 2ab\cos C$

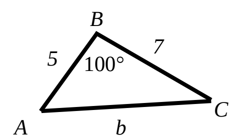
<table>
<tr><td>The Law of Cosines:</td><td>By placing the other two angles in standard position, we can find similar relationships for all the angles. The Law of Cosines is summarized as follows:

Law of Cosines
$a^2 = b^2 + c^2 - 2bc\cos A$
$b^2 = a^2 + c^2 - 2ac\cos B$
$c^2 = a^2 + b^2 - 2ab\cos C$

</td></tr>
<tr><td colspan="2">Define the following terms.</td></tr>
<tr><td>Unit Circle</td><td></td></tr>
<tr><td>Standard Position</td><td></td></tr>
<tr><td>Distance Formula</td><td></td></tr>
<tr><td colspan="2">Now, paraphrase and annotate the information on pages 1 and 2 and the top of this page.</td></tr>
</table>

We cannot use the Law of Sines to solve a triangle when the available information is two sides and the included angle (SAS) or the three sides (SSS). However, the Law of Cosines allows us to solve these types of problems.

Example 1:

Use the given side-angle-side (SAS) measurements to solve for ΔABC. Round to the nearest tenth.



1. Find the length of the third side.

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$b^2 = 7^2 + 5^2 - 2 \cdot 7 \cdot 5\cos 100°$$

$$b^2 \approx 86.2$$

$$b \approx 9.3$$

```
7²+5²-70cos(100)
         86.15537244
√Ans
         9.281991836
```

2. Use the Law of Sines to find an angle measure.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\sin A = \frac{a\sin B}{b}$$

$$\sin A = \frac{7\sin 100°}{9.3}$$

$$m\angle A = \operatorname{Sin}^{-1}\left(\frac{7\sin 100°}{9.3}\right)$$

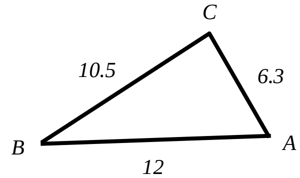
$$m\angle A \approx 47.8°$$

```
7sin(100)/9.3
       .7412531474
sin⁻¹(Ans)
       47.83827419
```

	3. Find the third angle measure. $47.8° + 100° + m\angle C \approx 180°$ $m\angle C \approx 180° - 47.8° - 100° \approx 32.2°$
***Exercise 1*:**	Use the given measurements to solve for ΔABC. Round to the nearest tenth. $b = 23, \qquad c = 18, \qquad m\angle A = 173$ The given information is side-angle-side (SAS).

Example 2:

Use the given side-side-side (SSS) measurements to solve for ΔABC. Round to the nearest tenth.



1. Find the measure of the largest angle, $\angle C$.

   $c^2 = a^2 + b^2 - 2ab\cos C$

   $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

   $\cos C = \dfrac{10.5^2 + 6.3^2 - 12^2}{2 \cdot 10.5 \cdot 6.3}$

   $\cos C \approx 0.0449$

   $m\angle C \approx \mathrm{Cos}^{-1}(0.0449)$

   $m\angle C \approx 87.4°$

```
10.5²+6.3²-12²
                5.94
Ans/(2*10.5*6.3)
         .0448979592
cos⁻¹(Ans)
         87.42667137
```

2. Find another angle measure.

   $b^2 = a^2 + c^2 - 2ac\cos B$

   Using a calculator as we did in step 1, we find

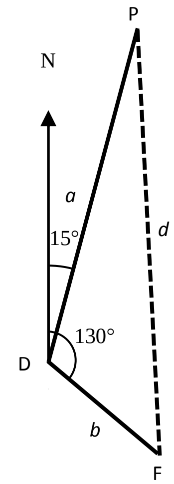
```
10.5²+12²-6.3²
            214.56
Ans/(2*10.5*12)
         .8514285714
cos⁻¹(Ans)
         31.63260937
```

   $m\angle B \approx 31.6°$

	3. Find the third angle measure. $m\angle A + 31.6 + 87.4 \approx 180$ $m\angle A \approx 61.0°$
***Exercise 2*:**	Use the given measurements to solve for ΔABC. Round to the nearest tenth. $a = 35, \qquad b = 42, \qquad c = 50.3$ The given information is side-side-side (SSS).

***Example 3*:**	A coast guard patrol boat and a fishing boat leave a dock at the same time. The patrol boat travels at a speed of 12 nautical miles per hour (12 knots) on a heading of 15° east of north. The fishing boat travels at a speed of 5 knots on a heading of 130° east of north. After 3 hours, the fishing boat sends a distress signal picked up by the patrol boat. If the fishing boat does not drift, and the patrol boat maintains its speed of 12 knots, how long will it take the patrol boat to reach the fishing boat?
***Step 1*:**	Read and reread the problem. Determine the following: 1. What is the problem asking? The number of hours the patrol boat needs to reach the fishing boat after receiving the distress call. 2. What information does the problem statement provide? a. The patrol boat's speed is 12 knots, and its direction is 15° east of north. b. The fishing boat's speed is 5 knots, and its direction is 130° east of north. c. The boats travel 3 hours before the distress call is sent.

Step 2: Let d be the distance between the patrol boat and the fishing boat after 3 hours. Draw a figure.



Step 3: Use the Law of Cosines to write an equation for d.

$$d^2 = a^2 + b^2 - 2ab\cos D$$

Where,

a is the distance between the dock and the patrol boat, and

b is the distance between the dock and the fishing boat.

Step 4: Find a, b, and $m\angle D$. Then solve the equation.

$$a = 3 \cdot 12 = 36$$

$$b = 3 \cdot 5 = 15$$

$$m\angle D = 130° - 15° = 115°$$

$$d^2 = 36^2 + 15^2 - 2 \cdot 36 \cdot 15 \cos 115°$$

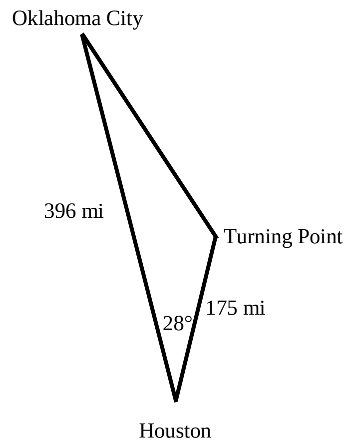
$$\approx 1977.4$$

$$d \approx 44.5 \text{ nautical miles}$$

Now, use the rate equation to find the time required to travel 44.5 nautical miles at 12 nautical miles per hour.

$$time = \frac{distance}{rate\ of\ travel}$$

$$t \approx \frac{44.5}{12} \approx 3.7$$

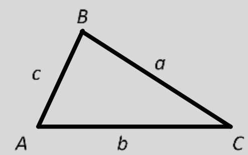
Step 5:	The patrol boat will take approximately 3.7 hours to reach the fishing boat after receiving the distress call. The patrol boat will have to travel farther after receiving the distress call than before receiving it. This answer is reasonable.
Exercise 3:	A pilot is flying from Houston to Oklahoma City, a distance of 396 miles. To avoid a thunder storm, the pilot flies 28° off the direct route for a distance of 175 miles. Then he makes a turn and flies straight to Oklahoma City. To the nearest mile, how much further than the direct route was the route taken by the pilot? 

***Heron's Formula*:**

Although it may have predated him by many centuries, the earliest known publication Heron's formula for calculating the area is attributed to Hero (or Heron) of Alexandria (c. 10 – 70 AD)[1].

Heron's Formula can be proved using the Law of Cosines.

Heron's Formula



Let s be half the perimeter of ΔABC.

$$s = \frac{1}{2}(a + b + c)$$

$$Area = \sqrt{s(s-a)(s-b)(s-c)}$$

***Example 4*:**

A blueprint shows a reception area that has triangular floor with sides measuring 22 ft, 30 ft, and 34 ft. What is the area of the floor to the nearest square foot?

1. Find the value of s.

$$s = \frac{1}{2}(a + b + c)$$

$$s = \frac{1}{2}(22 + 30 + 34)$$

$$s = 43$$

2. Find the area of the triangle.

$$area = \sqrt{s(s-a)(s-b)(s-c)}$$

$$area = \sqrt{43(43-22)(43-30)(43-34)}$$

$$area = \sqrt{43 \cdot 21 \cdot 13 \cdot 9} \approx 325\ ft^2$$

[1] http://en.wikipedia.org/wiki/Heron%27s_formula

	3. Check Use the Law of Cosines to find one of the angles. Then use SAS to find the area of the triangle. $c^2 = a^2 + b^2 - 2ab\cos C$ $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$ $\cos C = \dfrac{22^2 + 30^2 - 34^2}{2 \cdot 22 \cdot 30} \approx 0.173$ $m\angle C \approx \mathrm{Cos}^{-1}\, 0.173 \approx 80.1°$ Using SAS, $area = \dfrac{1}{2} ab \sin C$ $area \approx \dfrac{1}{2} \cdot 22 \cdot 30 \cdot \sin 80.1° \approx 325 ft^2$ The solution checks.
***Exercise 4*:**	The surface of a hotel swimming pool is shaped like a triangle with sides measuring 50 m, 28 m, and 30 m. What is the area of the pool's surface to the nearest square meter?
Class work:	P 734: 1-8
Homework:	P 735: 9-41 odd