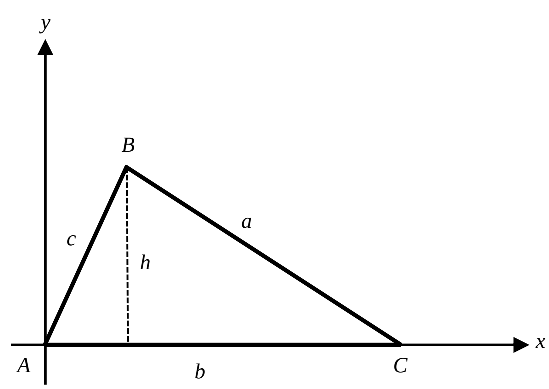
Transition to College Math

Name ________________________________

Period __________

Date: Unit **3: Trigonometry** Lesson **5: The Law of Sines**	Essential Question: **How can a sail maker use sine ratios to determine the amount of fabric needed to make a sail?**
Standard: G-SRT.10 G-SRT.11	Prove the Laws of Sines and Cosines and use them to solve problems. Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
***Objective*:**	Determine the area of a triangle given side-angle-side information. Use the Law of Sines to find the side lengths and angle measures of triangles. The student will demonstrate mastery by completing the exercises in these notes.
***Proof of the Law of Sines*:**	Draw a triangle *ABC*, with $\angle A$ in standard position. 
Summary	

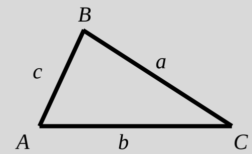
	The common practice is to use capital letters to represent angles and lower case letters to represent sides. Moreover, side a is opposite $\angle A$. Let K be the area of ΔABC. Then, $K = \frac{1}{2}(base)(height)$ $K = \frac{1}{2}bh$ We can find h using the sine of A. $\sin A = \frac{h}{c}$ $h = c \sin A$ Therefore, $K = \frac{1}{2}bc \sin A$ Notice that this formula is true regardless of whether $\angle A$ is acute or obtuse. Similarly, $K = \frac{1}{2}ac \sin B$ $K = \frac{1}{2}ab \sin C$ Since the area must be the same no matter how we find it, $\frac{1}{2}bc \sin A = \frac{1}{2}ac \sin B = \frac{1}{2}ab \sin C$ Finally, we divide everything by $\frac{1}{2}abc$. $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Law of Sines:

The Law of Sines

In any triangle ABC,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



Example 1:

Suppose a sail has two sides with lengths of 2.96 m and 2.13 m, and the angle between the two sides is 73°. To the nearest tenth of a square meter, what is the area of the sail?

From the previous page,

$$area = \frac{1}{2} bc \sin A$$

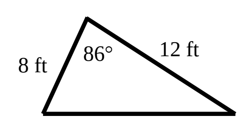
$$area = \frac{1}{2} 2.96 \cdot 2.13 \sin 73°$$

Using a calculator,

$$area \approx 3.0m^2$$

Exercise 1:

Find the area of the triangle. Round to the nearest tenth.



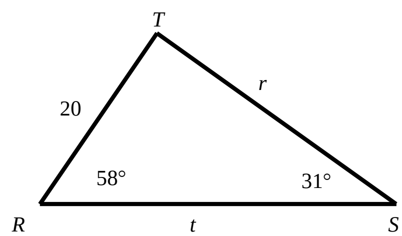
Solving Triangles:

The Law of Sines allows you to solve any triangle if you know either of the following:

1. Two angle measures and any side length.
   a. angle-side-angle (ASA)
   b. angle-angle-side (AAS)
2. Two side lengths and the measure of an angle that is ***not*** between them.

Example 2:

Solve the triangle. Round to the nearest tenth.



1. Find the third angle measure.

   $m\angle R + m\angle S + m\angle T = 180°$

   $m\angle T = 180° - 58° - 31°$

   $m\angle T = 91°$

2. Use the Law of Sines to find the unknown side lengths.

   $$\frac{\sin R}{r} = \frac{\sin S}{s}$$

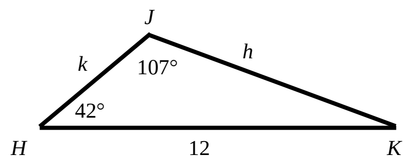
   $$r = \frac{\sin R}{\sin S} \cdot s = \frac{\sin 58°}{\sin 31°} 20 \approx 32.9$$

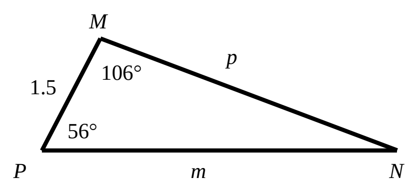
   Similarly.

   $$\frac{\sin T}{t} = \frac{\sin S}{s}$$

   $$t = \frac{\sin T}{\sin S} \cdot s = \frac{\sin 91°}{\sin 31°} 20 \approx 38.8$$

Exercise 2:	Solve the following triangles. Round to the nearest tenth.

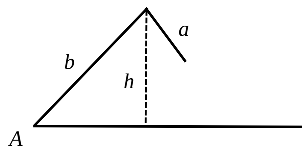




***Ambiguous Case*:** When you use the Law of Sines to solve a triangle for which you know side-side-angle information (SSA), zero, one, or two triangles may be possible. For this reason, SSA is called the ambiguous case.

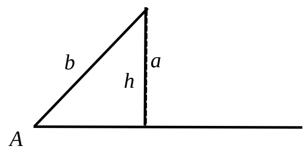
***$\angle A$ is Acute*:**

$a < h$



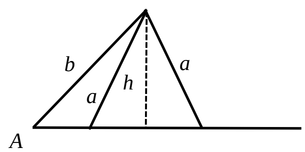
No triangle possible.

$a = h$



One triangle possible.

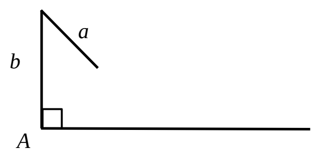
$a > h$



Two triangles possible

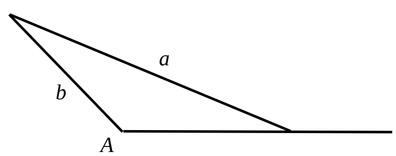
***$\angle A$ is Right or obtuse*:**

$a \leq b$



No triangle possible.

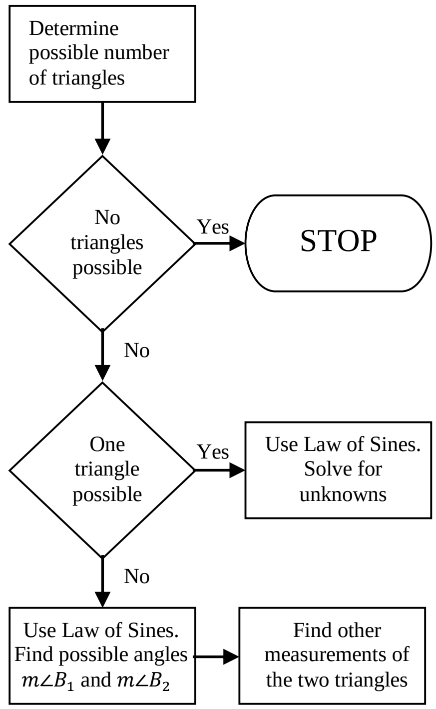
$a > b$



One triangle possible.

Solving a Triangle:

Given sides a and b, and $m\angle A$, Use the following flow chart:



Example 3:

Maggie is designing a mosaic by using triangular tiles of different shapes. Determine the number of triangles that Maggie can form using the measurements $a = 11\ cm$, $b = 17\ cm$, and $m\angle A = 30°$. Then solve the triangles.

1. Determine the number of possible triangles. Since $\angle A$ is acute, find h.

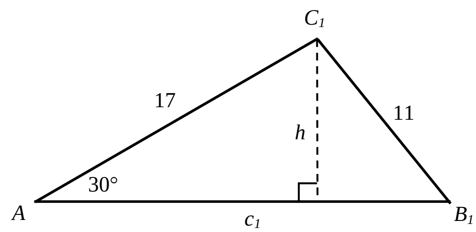
$$\sin 30° = \frac{opposite}{hypotenuse} = \frac{h}{17}$$

$$h = 17 \sin 30° = 8.5\ cm$$

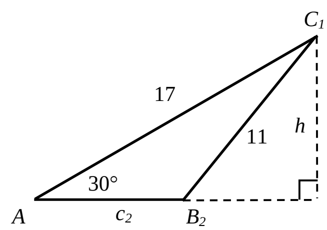
$$h < a < b$$

$\therefore$ two triangles are possible.

Triangle 1



Triangle 2



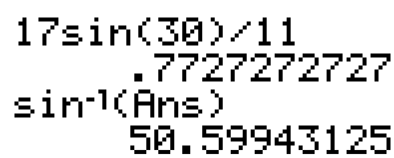
2. Determine $m\angle B_1$ and $m\angle B_2$.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 30°}{11} = \frac{\sin B}{17}$$

$$\sin B = \frac{17}{11}\sin 30° \approx 0.773$$

Let $\angle B_1$ represent the acute angle with a sine equal to 0.773. Use the inverse sine function on a calculator, to determine $\angle B_1$



$m\angle B_1 \approx 50.6°$

Let $\angle B_2$ represent the obtuse angle with a sine equal to 0.773.

$m\angle B_2 \approx 180° - 50.6° = 129.4°$

	3. Find the other unknown measures of the two triangles. a. Solve for $m\angle C_1$. $30° + 50.6° + \ m\angle C_1 = 180°$ $m\angle C_1 = 99.4°$ b. Solve for c_1. $\frac{\sin A}{a} = \frac{\sin C_1}{c_1}$ $\frac{\sin 30°}{11} = \frac{\sin 99.4°}{c_1}$ $c_1 = \frac{\sin 99.4°}{\sin 30°} 11 \approx 21.7\ cm$ c. Solve for $m\angle C_2$. $30° + 129.4° + \ m\angle C_2 = 180°$ $m\angle C_1 = 20.6°$ d. Solve for c_2. $\frac{\sin A}{a} = \frac{\sin C_2}{c_2}$ $\frac{\sin 30°}{11} = \frac{\sin 20.6°}{c_2}$ $c_2 = \frac{\sin 20.6°}{\sin 30°} 11 \approx 7.7\ cm$

***Exercise 3*:**	Determine the number of triangles that Maggie can form using the measurements $a = 10\ cm$, $b = 6\ cm$, and $m\angle A = 105°$. Then solve the triangles. Round to the nearest tenth.
Class work: p 726: 1-13	
Homework: p 727: 15-45 odd, 46	