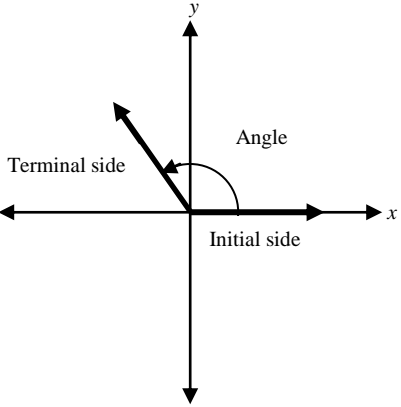


# Transition to College Math

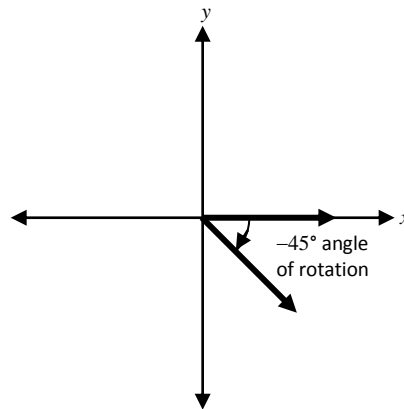
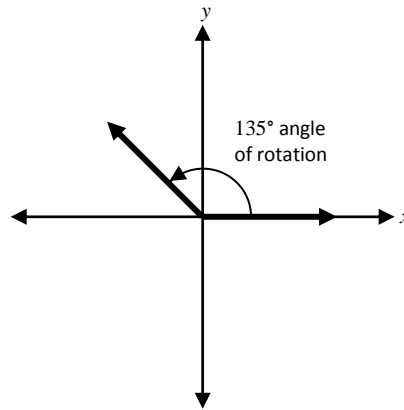
Name \_\_\_\_\_

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<p>Date:</p> <p>Unit 3: <b>Trigonometry</b></p> <p>Lesson 2: <b>Angles of Rotation</b></p>	<p>Essential Question: <b>What is the reference angle for an angle of <math>-15^\circ</math>?</b></p>
<p>Standard: F-TF.2</p>	<p>Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers</p>
<p><b>Learning Target:</b></p>          <p><b>Standard Position:</b></p>	<p>Draw angles in standard position. Determine the values of the trigonometric functions for an angle in standard position.</p> <p>Find <math>\sin(-135^\circ)</math></p> <p>In the previous lesson, we learned the trigonometric functions of acute angles in right triangles. In this lesson, we will extend our understanding of trigonometric functions to all angles.</p> <p>An angle is in standard position when its vertex lies on the origin of the coordinate plane and one ray is on the positive <math>x</math>-axis. The ray lying on the <math>x</math>-axis is the initial side, and the other ray is the terminal side.</p> 
<p>Summary</p>	

***Angle of Rotation:***

An angle of rotation is formed by keeping the initial side fixed and rotating the terminal side. If the terminal side rotates counterclockwise, the angle of rotation is positive. However, if the terminal side rotates clockwise, the angle of rotation is negative.



In your own words, define the following terms:

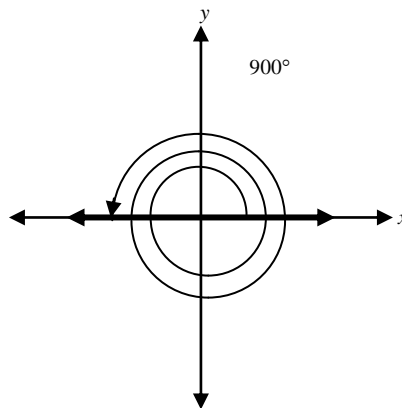
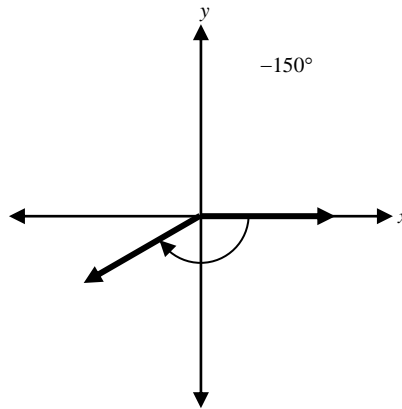
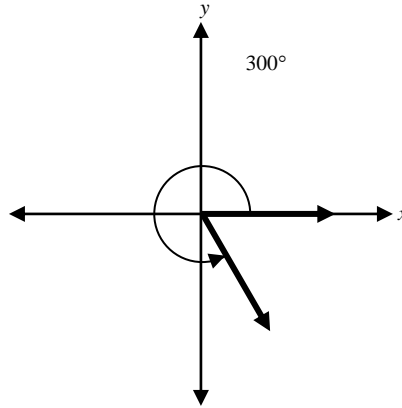
<i>Acute Angles</i>	
<i>Standard Position</i>	
<i>Vertex</i>	
<i>Coordinate Plane</i>	
<i>Origin</i>	
<i>Ray</i>	

In your own words, define the following terms:

<i>x-axis</i>	
<i>Initial Side</i>	
<i>Terminal Side</i>	
<i>Angle of Rotation</i>	
<i>Clockwise</i>	
<i>Counterclockwise</i>	

**Example 1:**

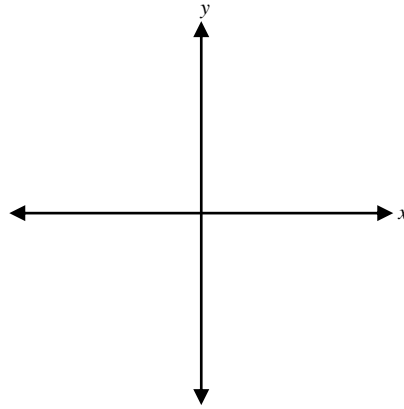
Draw angles in standard position.



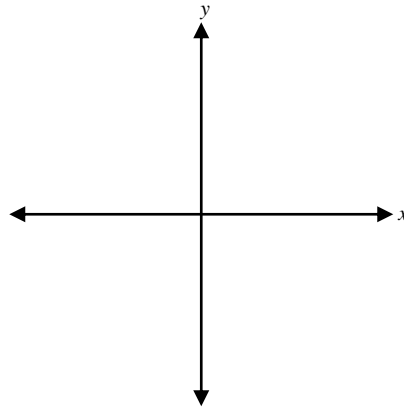
**Exercise 1:**

Draw angles with the given measure in standard position.

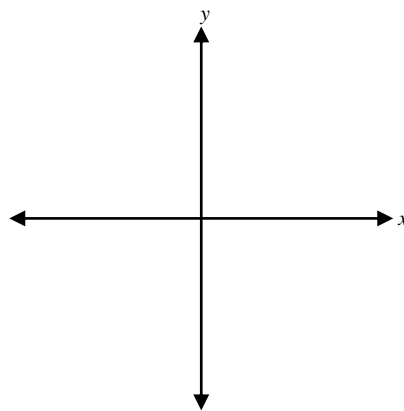
a.  $270^\circ$

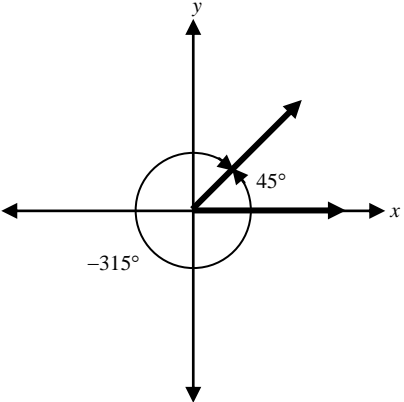


b.  $-810^\circ$



c.  $-315^\circ$



<p><b><i>Coterminal Angles:</i></b></p>	<p>Angles in standard position with the same terminal side are coterminal angles. For example, an angle measuring <math>45^\circ</math> is coterminal with an angle measuring <math>-315^\circ</math>.</p>  <p>Given an angle in standard position with measure <math>m</math> degrees, you can find another angle in standard position that is coterminal by rotating the terminal side an integral multiple of <math>360^\circ</math>. Specifically, all angles in standard position that have measures <math>m + 360n</math> where <math>m</math> is a degree measure and <math>n</math> is any integer are coterminal.</p>
<p><b><i>Coterminal Angles</i></b></p>	
<p><b><i>Angle Measure</i></b></p>	

**Example 2:**

Find an angle with positive measure and an angle with negative measure that are coterminal with the given angle.

a.  $\theta = 40^\circ$

$$40 + 360^\circ = 400^\circ$$

$$40 - 360^\circ = -320^\circ$$

Angles with measures of  $400^\circ$  and  $-320^\circ$  are coterminal with an angle with a  $40^\circ$  angle.

b.  $\theta = 380^\circ$

$$380 - 360^\circ = 20^\circ$$

$$380 - 720^\circ = -340^\circ$$

Angles with measures of  $20^\circ$  and  $-340^\circ$  are coterminal with an angle with a  $380^\circ$  angle.

**Exercise 2:**

Find an angle with positive measure and an angle with negative measure that are coterminal with the given angle.

a.  $\theta = 76^\circ$

b.  $-1000^\circ$

c.  $-52^\circ$

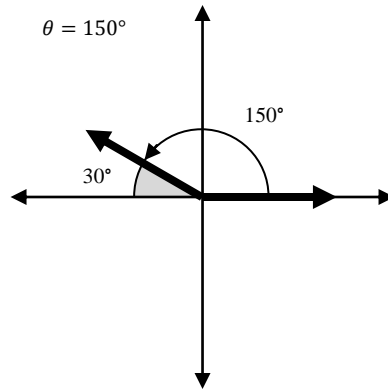


**Reference Angle:**

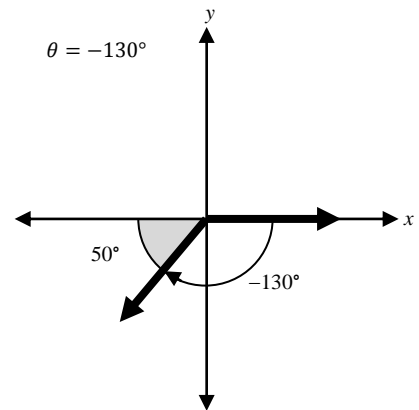
For an angle in standard position, the reference angle is the positive, **acute angle** formed by the terminal side and the **x-axis**.

**Example 3:**

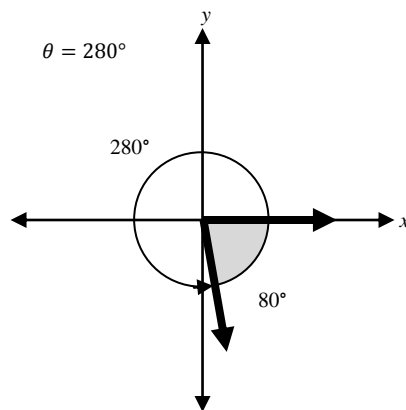
Find the measure of the reference for each given angle.



The measure of the reference angle is  $30^\circ$ .



The measure of the reference angle is  $50^\circ$ .



The measure of the reference angle is  $80^\circ$ .

***Exercise 3:***

Find the measure of the reference for each given angle.

a.  $\theta = 105^\circ$

b.  $\theta = -115^\circ$

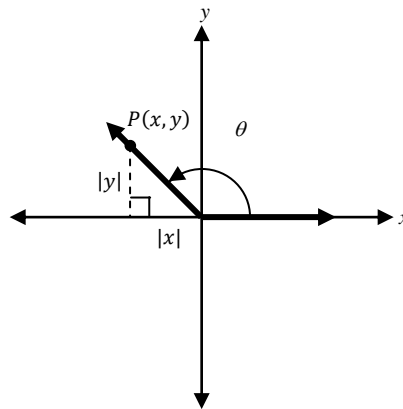
c.  $\theta = 310^\circ$

***Finding Values of the Trigonometric Functions:***

You can use the reference angle to find the values of the trigonometric functions for angles measuring less than  $0^\circ$  or greater than  $90^\circ$ .

To find the trigonometric functions of an angle in standard position, first select a point that lies on the terminal side. This point cannot lie on the origin, but any other point will do. Suppose this point,  $P$ , has coordinates  $(x, y)$ . Use the Pythagorean Theorem to calculate the distance of  $P$  from the origin.

$$r = \sqrt{x^2 + y^2}$$



The sine, cosine, and tangent functions are defined as follows:

$$\sin \theta = \frac{y}{r}$$

$$\cos \theta = \frac{x}{r}$$

$$\tan \theta = \frac{y}{x} \quad x \neq 0$$

$$x, y, r \in \mathbb{R}, r > 0$$

Notice that the tangent is undefined when the terminal side of an angle in standard position lies on the  $y$ -axis. Moreover, the cotangent is undefined when the terminal side of an angle in standard position lies on the  $x$ -axis.

**Example 4:**

Find the exact values of the six trigonometric functions for an angle in standard position with measure  $\theta$  if the point  $P(4, -5)$  lies on the terminal side of the angle.

1. Use the Pythagorean Theorem to calculate the distance between P and the origin.

$$r = \sqrt{4^2 + (-5)^2} = \sqrt{41}$$

2. Find the sine, cosine, and tangent.

$$\sin \theta = \frac{y}{r} = \frac{-5}{\sqrt{41}} = -\frac{5\sqrt{41}}{41}$$

$$\cos \theta = \frac{x}{r} = \frac{4}{\sqrt{41}} = \frac{4\sqrt{41}}{41}$$

$$\tan \theta = \frac{y}{x} = \frac{-5}{4} = -\frac{5}{4}$$

3. Use the reciprocals to find the cosecant, secant, and cotangent.

$$\csc \theta = \frac{1}{\sin \theta} = -\frac{\sqrt{41}}{5}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{\sqrt{41}}{4}$$

$$\cot \theta = \frac{1}{\tan \theta} = -\frac{4}{5}$$

**Exercise 4:**

Find the exact values of the six trigonometric functions for an angle in standard position with measure  $\theta$  if the point  $P(-3, 6)$  lies on the terminal side of the angle.

**Class work:** p 703 Guided Practice: 1-25

**Homework:** p 703 Practice and Problem Solving: 26-68, 72, 73