Transition to College Math

Name ________________________________

Period __________

<table>
<tr><td>Unit 1: Series and Sequences

Lesson 7: Mathematical Induction</td><td>Essential Question: How is a repeating decimal like an infinite geometric series</td></tr>
<tr><td>Standard: F-BF.2</td><td></td></tr>
<tr><td>Learning Target:

Example 1:
Repeating Decimal:</td><td>Students will learn to use mathematical induction to prove statements. 80% of the students will be able to prove that $0.\overline{123} = \frac{41}{333}$.

A repeating decimal can be written as an infinite geometric series. For example,

$$0.1212121212\cdots = 0.12 + 0.00120 + 0.000012 + \cdots$$
$$= \sum_{i=1}^{\infty} 0.12\left(\frac{1}{100}\right)^{i-1}$$
This is a converging infinite geometric series with.

$a_1 = 0.12$

$r = \frac{1}{100}$

$\therefore$

$$S = \frac{a_1}{1-r} = \frac{0.12}{1-0.01} = \frac{0.12}{0.99} = \frac{12}{99} = \frac{4}{33}$$</td></tr>
<tr><td colspan="2">Summary

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	In simplest terms, $$0.1212121212 = \frac{4}{33}$$
***Exercise*: 1:**	Write as a fraction in simplest form. a. $0.11111\cdots$ b. $0.\overline{142857}$

***Mathematical Induction*:**	In a previous lesson, we developed a formula to find the sum of the first *n* Natural numbers. $$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$ We could use mathematical induction to prove this formula. A proof by mathematical induction requires three steps. These steps are outlined in the box below: To prove a statement is true for all natural numbers n, 1. Show that the statement is true for $n = 1$. 2. Assume that the statement is true for a natural number n. 3. Prove that the statement is true for the natural number $n + 1$.
***Example 2*:**	Use mathematical induction to prove that the sum of the first *n* natural numbers is, $$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$ 1. Show that the statement is true for $n = 1$. $$1 = \frac{n(n+1)}{2} = \frac{1(1+1)}{2} = \frac{2}{2} = 1$$ The formula is true for $n = 1$ (the base case.) 2. Assume that the statement is true for a natural number n. Assume, $$1 + 2 + 3 + \cdots n = \frac{n(n+1)}{2}$$

	3. Prove that the statement is true for the natural number $n+1$. $1+2+3+\cdots n+(n+1)=\frac{n(n+1)}{2}+(n+1)$ $=\frac{n(n+1)}{2}+\frac{2(n+1)}{2}=\frac{n(n+1)+2(n+1)}{2}$ $=\frac{(n+1)(n+2)}{2}$ $Q.E.D$
***Exercise 2*:**	a. Use mathematical induction to prove that the sum of the first *n* odd numbers is, $1+3+5+7+\cdots(2n-1)=n^2$

	b. Use mathematical induction to prove that the sum of the squares of the first n natural numbers is, $$1 + 4 + 9 + 16 + \cdots n^2 = \frac{n(n+1)(2n+1)}{6}$$

***Using Counterexamples*:**	Mathematical statements that seem true may be false. One way to prove them false is to find a counterexample.
***Example 3*:**	Identify a counterexample to disprove $2^x \geq x^2$, where x is a real number. Let $x = -1$ $2^{-1} = \frac{1}{2} < 1 = (-1)^2$
***Exercise 3*:**	Identify a counterexample to disprove $\frac{a^2}{2} \leq 2a + 1$, where a is a real number.