Transition to College Math

Name ______________________________

Date ____________ Period _________

Unit **1: Series and Sequences** Lesson **6: Infinite Geometric Series**	Essential Question: **Why might the notation for a partial sum S_n change to the notation S for an infinite geometric series?**
Standard: F-BF.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
Learning Target:	The student will learn to find the sum of an infinite geometric series. 80% of the students will be able to find, $$\sum_{i=1}^{\infty} 60\left(\frac{1}{10}\right)^i$$
Weekend Box Office Sales for Spider Man:	When the movie *Spider-Man* was released, total box office sales (in millions of dollars) for the first three weekends are given in the table below.

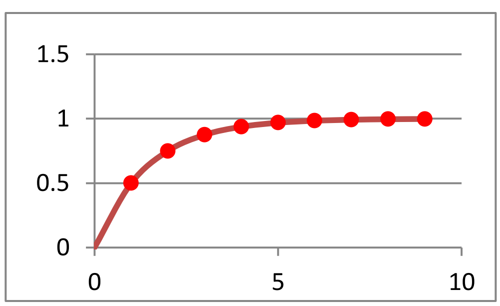
Weekend	Box Office
1	114.9
2	71.4
3	45.0

Find a plausible rule and use it to predict the weekend sales for weeks 4 and 5.

Write and evaluate a series summation for the first 5 weeks.

Suppose the series continued infinitely. What would the sum be?

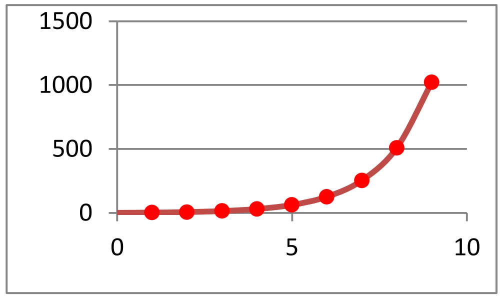
Summary

Limits:	A ***limit*** is the value that a function approaches when the argument of the function approaches some preselected value. For example when the value of *x* becomes very large (approaching infinity), the value of the function $f(x) = \frac{1}{x}$ becomes very nearly zero. In fact, by making the value sufficiently large, we can make the function's value as close to zero as we choose. We say that the limiting value of this function as x becomes arbitrarily large is zero. We write this symbolically as, $$\lim_{x\to\infty} f(x) = 0$$
Infinite Geometric Series:	An infinite geometric series is the value that a partial geometric series approaches in the limit where n becomes very large. If a partial sum of a geometric series is, $$S_n = \sum_{i=1}^{n} a_1 \cdot r^{i-1}$$ then, $$S = \lim_{n\to\infty} S_n = \sum_{i=1}^{\infty} a_1 \cdot r^{i-1}$$ Consider the partial sum of the following geometric series: $$S_n = \sum_{i=1}^{n} \left(\frac{1}{2}\right)^i = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \left(\frac{1}{2}\right)^n$$ A graph of partial sums where *n* is the domain variable would look like this,  The partial sum appears to approach a value of 1.0 as *n* increases.

Now, consider the partial sum of the following geometric series:

$$S_n = \sum_{i=1}^{n} (2)^i = 2 + 4 + 8 + \cdots + 2^n$$

A graph of partial sums where *n* is the domain variable would look like this,



The partial sum appears to grow without limit as *n* increases.

Convergence:

If a finite geometric series approaches a finite number as *n* becomes very large, the series is said to ***converge***. The first series above converges.

If a finite geometric series approaches infinity (or negative infinity) as *n* becomes very large, the series is said to ***diverge***. The second series above diverges.

Recall the formula for finding the partial sum of a geometric series:

$$S_n = a_1 \frac{1 - r^n}{1 - r}$$

What would happen if *n* became very large? Clearly $r \neq 1$. If that were the case, S_n would be undefined. That leaves us with two cases:

I. $|r| < 1$ and

II. $|r| > 1$.

Let's consider each case:

I. If *n* is a positive integer, then $|r^n| < 1$. Moreover, as n becomes very large, $|r^n|$ becomes very close to zero. In symbolic terms,

$$\lim_{n \to \infty} (|r^n|) = 0$$

Therefore,

$$S = a_1 \frac{1}{1 - r}$$

II. If *n* is a positive integer, then $|r^n| > 1$. Moreover, as n becomes very large, $|r^n|$ becomes very large. In symbolic terms,

$$\lim_{n \to \infty} (|r^n|) = \infty$$

Therefore, the infinite geometric sum is undefined. In short, the value of *r* determines whether or not the infinite geometric series converges:

$|r| < 1 \Rightarrow S$ converges.

$|r| > 1 \Rightarrow S$ diverges.

Example 1:	Determine whether each geometric series converges or diverges. a. $20 + 24 + 28.8 + 34.56 + \cdots$ $r = \frac{24}{20} = 1.2$ $\lvert r \rvert \geq 1$ The series diverges and does not have a sum. b. $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \cdots$ $r = \frac{\frac{1}{3}}{1} = \frac{1}{3}$ $\lvert r \rvert < 1$ The series converges and has a sum.

***Exercise 1*:**	Determine whether each geometric series converges or diverges. a. $32 + 16 + 8 + 4 + 2 + \cdots$ b. $\frac{2}{3} + 1 + \frac{3}{2} + \frac{9}{4} + \frac{27}{8} + \cdots$
***Sum of an Infinite Geometric Series*:**	As we noted above, The sum of an infinite geometric series S with common ratio r and $\lvert r\rvert < 1$ is, $$S = \frac{a_1}{1 - r}$$ Where a_1 is the first term.

***Example 2*:**	Find the sum of each infinite geometric series, if it exists. a. $5 + 4 + 3.2 + 2.56 + \cdots$ $r = \frac{4}{5} = 0.8$ $\lvert r \rvert < 1$ The series converges and does have a sum. $S = \frac{a_1}{1 - r}$ $S = \frac{5}{1 - 0.8} = \frac{5}{0.2} = 25$ b. $\sum_{i=1}^{\infty} \left(\frac{1}{3}\right)^{i-1}$ $a_1 = \left(\frac{1}{3}\right)^{1-1} = \left(\frac{1}{3}\right)^{0} = 1$ $r = \frac{1}{3}$ $\lvert r \rvert < 1$ The series converges and does have a sum. $S = \frac{a_1}{1 - r}$ $S = \frac{1}{1 - \frac{1}{3}} = \frac{1}{\frac{2}{3}} = \frac{3}{2}$

Exercise 2:	Find the sum of each infinite geometric series, if it exists. a. $2 + 1 + \frac{1}{2} + \frac{1}{4} + \cdots$ b. $\sum_{i=1}^{\infty} -3(.2)^{i-1}$