Transition to College Math Name ______________________________

Period __________

Date: Unit: 1: **Series and Sequences** Lesson: **5: Arithmetic and Geometric Series**	Essential Question: **Suppose you purchase a collectable for \$100. It could increase in value by \$15 per year or by 10% per year. Which would be better for you? Explain.**
Standard: A.SSE.4	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.
Learning Target: ***Mortgage Payment***	Find the sums of arithmetic and geometric series. 80% of the students will find S_{10} for the sequence $a_n = 3n - 5$ Suppose you want to buy a house. The bank will give you a 30 year loan for \$100,000. The interest rate is 6%. If you make monthly payments, how much will your monthly mortgage payment be? 
Summary	

Arithmetic Series:	The sum of the terms of an arithmetic sequence is an ***arithmetic series***. The notation for an arithmetic series is, $$S_n = \sum_{i=1}^{n} a_i = a_1 + (a_1 + d) + (a_1 + 2d) + \cdots + a_n$$ Where a_i is the i^{th} term of an arithmetic sequence.
Partial Sum:	S_n is called the ***partial sum*** of the arithmetic series. You can derive the formula for the partial sum by adding S_n to itself in reverse order. $$S_n + S_n$$ $$= a_1 + (a_1 + d) + (a_1 + 2d) + (a_1 + 3d) + \cdots + a_n$$ $$+ a_n + (a_n - d) + (a_n - 2d) + (a_n - 3d) + \cdots + a_1$$ $$= (a_1 + a_n) + (a_1 + a_n) + (a_1 + a_n) + \cdots + (a_1 + a_n)$$ $$2S_n = n(a_1 + a_n)$$ $$S_n = n\frac{(a_1 + a_n)}{2}$$ The partial sum S_n of the first n terms of an arithmetic series $a_1 + a_2 + \cdots + a_n$ is given by $$S_n = n\frac{(a_1 + a_n)}{2}$$ where a_1 is the first term, and a_n is the n^{th} term.

Geometric Series:

The sum of the terms of a geometric sequence is a ***geometric series***. The notation for a geometric series is,

$$S_n = \sum_{i=1}^{n} a_i = a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \cdots + a_1 r^{n-1}$$

Where a_i is the i^{th} term of a geometric sequence.

Partial Sum:

S_n is called the ***partial sum*** of the geometric series.

You can derive the formula for the partial sum by subtracting the product of r and S_n from S_n.

$$S_n - S_n r$$

$$= a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \cdots + a_1 r^{n-1}$$
$$- a_1 r - a_1 r^2 - a_1 r^3 - \cdots - a_1 r^{n-1} - a_1 r^n$$

$$S_n(1-r) = a_1(1-r^n)$$

$$S_n = a_1\left(\frac{1-r^n}{1-r}\right) = \sum_{i=1}^{n} a_i\, r^{i-1}$$

> The partial sum S_n of the first n terms of a geometric series $a_1 + a_2 + \cdots + a_n$ is given by
>
> $$S_n = a_1\left(\frac{1-r^n}{1-r}\right), r \neq 1$$
>
> where a_1 is the first term, and r is the common ratio.

Example 1:

Find the indicated sum for each geometric series.

a. S_7 for $3 - 6 + 12 - 24 + \cdots$

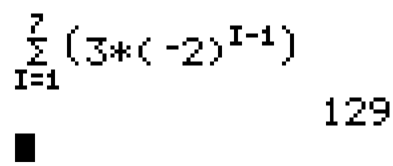
1. Find the common ratio

$$r = \frac{a_2}{a_1} = \frac{-6}{3} = -2$$

2. Find S_7 with $a_1 = 3$, $r = -2$ and $n = 7$.

$$S_n = a_1\left(\frac{1 - r^n}{1 - r}\right) = 3\left(\frac{1 - (-2)^7}{1 - (-2)}\right)$$
$$= 3\left(\frac{1 - (-128)}{3}\right) = 129$$

Check using a graphing calculator.



b.

$$\sum_{i=1}^{5}\left(\frac{1}{3}\right)^{i-1}$$

1. Find the first term

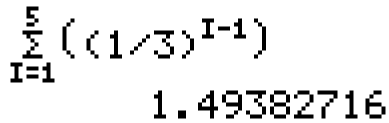
$$a_1 = \left(\frac{1}{3}\right)^{1-1} = \left(\frac{1}{3}\right)^{0} = 1$$

2. Find S_5,

$$S_n = a_1\left(\frac{1 - r^n}{1 - r}\right) = 1\left(\frac{1 - \left(\frac{1}{3}\right)^5}{1 - \left(\frac{1}{3}\right)}\right) = \left(\frac{1 - \left(\frac{1}{243}\right)}{\frac{2}{3}}\right)$$

$$= \frac{242}{243} \cdot \frac{3}{2} = \frac{121}{81} \cong 1.49$$

Check using a graphing calculator.



***Exercise 1*:**

Find the indicated sum for each geometric series.

a.

S_6 for $2 + 1 + \frac{1}{2} + \frac{1}{4} + \cdots$

b.

$$\sum_{i=1}^{5} -3(2)^{i-1}$$

Example 2:	The Wimbledon Ladies Singles Championship begins with 128 players. The players compete until there is 1 winner. How many matches must be scheduled in order to complete the tournament?

1. Write a sequence.

   Let $n =$ the number of rounds.

   $a_n =$ the number of matches played in the n^{th} round.

   $S_n =$ the total number of matches played through n rounds.

   $$a_n = 64\left(\frac{1}{2}\right)^{n-1}$$

2. The tournament is completed when $a_n = 1$.

   $$\frac{1}{64} = \left(\frac{1}{2}\right)^{n-1}$$

   $$\frac{1}{64} = \left(\frac{1}{2}\right)^{6} = \left(\frac{1}{2}\right)^{n-1}$$

   $$6 = n - 1$$

   $$n = 7$$

3. Find the total number of matches after 7 rounds1.

   $$S_7 = 64\left(\frac{1-\left(\frac{1}{2}\right)^7}{1-\left(\frac{1}{2}\right)}\right) = 127$$

***Exercise 2*:**	A 6-year lease states that the annual rent for an office space is $84,000 the first year and will increase by 8% each additional year of the lease. What will the total rent expense be for the 6 year lease?