Transition to College Math Name ________________________________

Period __________

Date: Unit 1: **Sequences and Series** Lesson 4: **Sigma Notation**	Essential Question: **What is the difference between a sequence and a series?**
Standard: F.IF.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
Learning Target:	80% of the students will be able to evaluate the following sum: $\sum_{i=1}^{15}(2i+3)$
Series:	You learned how to find the n^th^ term of a sequence. Often we are also interested in the sum of a certain number of terms in a sequence. A ***series*** is the sum of a certain number of terms in a sequence. Some examples are shown in the following table.
Partial Sums:	Many sequences are infinite and do not have defined sums. Therefore, we often find ***partial sums***. A partial sum is indicated by S_n, and it is the sum of the first n terms in the sequence.
Sigma Notation:	We often use sigma notation to represent a series. This notation uses the upper case Greek letter sigma (Σ) to indicate the sum of a sequence. The diagram on the next page depicts basic sigma notation.
Summary	

Sequence	$1, 2, 3, 4$	$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}$
Series	$1+2+3+4$	$\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}$

	$$\sum_{i=1}^{n} a_i$$ The lower equation (in this case, $i = 1$) indicates the first term of the sequence. The top number (n) indicates the last term of the sequence. The expression on the right (a_i) is the explicit rule for the i^{th} term of the sequence.
Example 1:	Write each series in sigma notation. a. $3 + 6 + 9 + 12 + 15$ Find a rule for the i^{th} term of the sequence $a_i = 3i$ Write the sigma notation for the first 5 terms $\sum_{i=1}^{5} 3i$ b. $\frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \frac{1}{32} - \frac{1}{64}$ Find a rule for the i^{th} term of the sequence $a_i = \frac{1}{2}\left(-\frac{1}{2}\right)^{i-1} = \frac{1}{2}(-1)^{i-1}\left(\frac{1}{2}\right)^{i-1} = (-1)^{i-1}\left(\frac{1}{2}\right)^{i}$ Write the sigma notation for the first 6 terms $\sum_{i=1}^{6} (-1)^{i-1}\left(\frac{1}{2}\right)^{i}$

***Alternating Signs*:**	For sequences with alternating signs, use the following: 1. $a_1 > 0 \Rightarrow use\ (-1)^{i+1}$ 2. $a_1 < 0 \Rightarrow use\ (-1)^{i}$
***Exercise 1*:**	Write each series in sigma notation. a. $-2+4-6+8-10+12$ b. $\frac{2}{4}+\frac{2}{9}+\frac{2}{16}+\frac{2}{25}+\frac{2}{36}$
***First Value not 1*:**	A sequence always begins with $i=1$; nevertheless, a series may begin with an index value other than $i=1$. For example, see example 2a.
***Example 2*:**	Expand and evaluate the following: a. $\sum_{i=3}^{6}\frac{1}{2^i}$ $\sum_{i=3}^{6}\frac{1}{2^i}=\frac{1}{2^3}+\frac{1}{2^4}+\frac{1}{2^5}+\frac{1}{2^6}$ $=\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\frac{1}{64}$ $=\frac{8}{64}+\frac{4}{64}+\frac{2}{64}+\frac{1}{64}=\frac{15}{64}$

	b. $$\sum_{i=1}^{4}(10-i^2)$$ $$\sum_{i=1}^{4}(10-i^2)$$ $$= (10-1^2)+(10-2^2) + (10-3^2)+(10-4^2)$$ $$= 9+6+1+(-6) = 10$$
***Exercise 2*:**	Expand and evaluate the following: a. $$\sum_{i=1}^{4}(2i-1)$$ b. $$\sum_{i=1}^{5}-5(2)^{i-1}$$

Formulas:	Finding the sum of a sequence with many terms can be tedious. Let's derive formulas for some common series.
Constant Series:	In a constant sequence, each term has the same value. Suppose each term in a sequence has the same value: $a_i = A, \qquad$ where A is a constant. Then the sum of the first n terms is, $$\sum_{i=1}^{n} a_i = \sum_{i=1}^{n} A = nA$$
Linear Series:	A linear series derives from am arithmetic sequence. Consider the arithmetic sequence that is the natural numbers, $1, 2, 3, 4, \cdots$. Find the sum of the first 10 natural numbers. $$\sum_{i=1}^{10} i = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$$ Consider what happens when we rearrange the terms. $$\sum_{i=1}^{10} i = (1 + 10) + (2 + 9) + (3 + 8) + (4 + 7) + (5 + 6)$$ $$\sum_{i=1}^{10} i = 11 + 11 + 11 + 11 + 11 = 5(11) = 55$$ Notice that 5 is half the number of terms, and 11 is the sum of the first and last terms. This suggests that the sum of a linear series is, $$\sum_{1}^{n} i = \frac{n}{2}(1 + n) = \frac{n(n + 1)}{2}$$
Quadratic Series:	Similar methods help us to find the sum of a quadratic sequence, $$\sum_{1}^{n} i^2 = \frac{n(n + 1)(2n + 1)}{6}$$

Summation Formulas:	***Constant Series*** $$\sum_{i=1}^{n} a_i = \sum_{i=1}^{n} A = nA$$ ***Linear Series*** $$\sum_{1}^{n} i = \frac{n(n+1)}{2}$$ ***Quadratic Series*** $$\sum_{1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$
Example 3:	Evaluate each series, a. $$\sum_{i=5}^{10} 8$$ Use the summation formula. Notice there are 6 terms; therefore, $$\sum_{i=5}^{10} 8 = 6(8) = 48$$ b. $$\sum_{i=1}^{5} i$$ Use the summation formula. Notice there are 5 terms, and the sum of the first and last terms is 6. Therefore, $$\sum_{i=1}^{5} i = \frac{5(5+1)}{2} = \frac{5(6)}{2} = 15$$

	c. $$\sum_{i=1}^{7} i^2$$ Use the summation formula. Notice there are 7 terms; therefore, $$\sum_{i=1}^{7} i^2 = \frac{7(7+1)(2 \cdot 7+1)}{6} = \frac{7 \cdot 8 \cdot 15}{6} = 140$$
Exercise 3:	Evaluate each series, a. $$\sum_{i=1}^{60} 4$$ b. $$\sum_{i=1}^{15} i$$

c.

$$\sum_{i=1}^{10} i^2$$

***Example 4*:**

Ricky is building a house of cards. He wants the house to have as many stories as possible. With a deck of 52 cards, how many stories will Ricky's house have?

1. Understand the Problem

The answer will be the number of rows in the card house. List the important information:

- He has 52 cards.
- The house should have as many stories as possible.

2. Make a Plan

- Make a diagram of the house.
- Find a pattern for the number of cards in each story.
- Write a series and evaluate it.

3. Solve

Make a table and a diagram.

Row	1	2	3	4
Diagram	/\	/▽\	/▽▽\	/▽▽▽\
Cards	2	5	8	11

	The number of cards increases by 3 in each row. Write a series to represent the total number of cards in *n* rows. $\sum_{i=1}^{n}(3i-1)$ It is important to recognize that the sigma notation is addition. Therefore, we can use the associative and distributive properties to rewrite this expression as follows: $\sum_{i=1}^{n}(3i-1) = 3\sum_{i=1}^{n} i - \sum_{i=1}^{n} 1$ The first sum is a linear series, and the second sum is a constant series. Therefore, $\sum_{i=1}^{n}(3i-1) = 3\frac{n(n+1)}{2} - n \cdot 1 = \frac{3n(n+1)}{2} - \frac{2n}{2}$ $= \frac{3n^2+3n-2n}{2}$ $= \frac{3n^2+n}{2}$ Now, we set this equal to the total number of cards available (52) and solve for *n*. $\frac{3n^2+n}{2} = 52$ $3n^2+n-104 = 0$ Using the quadratic formula, we find, $n \cong -6.06, 5.72$ We reject the negative answer and recognize that we cannot have a fractional answer. Therefore, we cannot have more than 5 stories. We evaluate the series with $n = 5$.

	$$\frac{3 \cdot 5^2 + 5}{2} = 40$$
***Exercise 4*:**	A flexible garden hose is coiled for storage. Each subsequent loop is 6 inches longer than the preceding loop, and the innermost loop is 35 inches long. If there are 6 loops, how long is the hose?
Class work:	p 638: 1-12
Homework:	p 638 ff: 13-51 odd