Transition to College Math Name ________________________________

Period __________

Date: Unit 1: **Sequences and Series** Lesson 4: **Sigma Notation**	Essential Question: **What is the difference between a sequence and a series?**
Standard: F-IF.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
***Learning Target*:**	80% of the students will be able to evaluate the following sum: $\sum_{i=1}^{15}(2i+3)$
***Series*:**	You learned how to find the n^th term of a sequence. Often we are also interested in the sum of a certain number of terms in a sequence. A ***series*** is the sum of a certain number of terms in a sequence. Some examples are shown in the following table.
***Partial Sums*:**	Many sequences are infinite and do not have defined sums. Therefore, we often find ***partial sums***. A partial sum is indicated by S_n, and it is the sum of the first n terms in the sequence.
***Sigma Notation*:**	We often use sigma notation to represent a series. This notation uses the upper case Greek letter sigma (Σ) to indicate the sum of a sequence. The diagram on the next page depicts basic sigma notation.

Sequence	$1, 2, 3, 4$	$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}$
Series	$1+2+3+4$	$\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}$

Summary

	$$\sum_{i=1}^{n} a_i$$ The lower equation (in this case, $i = 1$) indicates the first term of the sequence. The top number (n) indicates the last term of the sequence. The expression on the right (a_i) is the explicit rule for the i^{th} term of the sequence.
Example 1:	Write each series in sigma notation. a. $3 + 6 + 9 + 12 + 15$ Find a rule for the i^{th} term of the sequence $a_i = 3i$ Write the sigma notation for the first 5 terms $\sum_{i=1}^{5} 3i$ b. $\frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \frac{1}{32} - \frac{1}{64}$ Find a rule for the i^{th} term of the sequence $a_i = \frac{1}{2}\left(-\frac{1}{2}\right)^{i-1} = \frac{1}{2}(-1)^{i-1}\left(\frac{1}{2}\right)^{i-1} = (-1)^{i-1}\left(\frac{1}{2}\right)^{i}$ Write the sigma notation for the first 6 terms $\sum_{i=1}^{6} (-1)^{i-1}\left(\frac{1}{2}\right)^{i}$

***Alternating Signs*:**	For sequences with alternating signs, use the following: 1. $a_1 > 0 \Rightarrow use\ (-1)^{i+1}$ 2. $a_1 < 0 \Rightarrow use\ (-1)^{i}$
***Exercise 1*:**	Write each series in sigma notation. a. $-2 + 4 - 6 + 8 - 10 + 12$ b. $\frac{2}{4} + \frac{2}{9} + \frac{2}{16} + \frac{2}{25} + \frac{2}{36}$
***First Value not 1*:**	A sequence always begins with $i = 1$; nevertheless, a series may begin with an index value other than $i = 1$. For example, see example 2a.
***Example 2*:**	Expand and evaluate the following: a. $\sum_{i=3}^{6} \frac{1}{2^i}$ $\sum_{i=3}^{6} \frac{1}{2^i} = \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^5} + \frac{1}{2^6}$ $= \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64}$ $= \frac{8}{64} + \frac{4}{64} + \frac{2}{64} + \frac{1}{64} = \frac{15}{64}$

	b. $$\sum_{i=1}^{4}(10-i^2)$$ $$\sum_{i=1}^{4}(10-i^2)$$ $$= (10-1^2)+(10-2^2) + (10-3^2)+(10-4^2)$$ $$= 9+6+1+(-6) = 10$$
***Exercise 2*:**	Expand and evaluate the following:
	a. $$\sum_{i=1}^{4}(2i-1)$$
	b. $$\sum_{i=1}^{5}-5(2)^{i-1}$$

***Formulas*:**	Finding the sum of a sequence with many terms can be tedious. Let's derive formulas for some common series.
***Constant Series*:**	In a constant sequence, each term has the same value. Suppose each term in a sequence has the same value: $a_i = A,$ where A is a constant. Then the sum of the first n terms is, $\sum_{i=1}^{n} a_i = \sum_{i=1}^{n} A = nA$
***Linear Series*:**	A linear series derives from am arithmetic sequence. Consider the arithmetic sequence that is the natural numbers, $1, 2, 3, 4, \cdots$. Find the sum of the first 10 natural numbers. $\sum_{i=1}^{10} i = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$ Consider what happens when we rearrange the terms. $\sum_{i=1}^{10} i = (1 + 10) + (2 + 9) + (3 + 8) + (4 + 7) + (5 + 6)$ $\sum_{i=1}^{10} i = 11 + 11 + 11 + 11 + 11 = 5(11) = 55$ Notice that 5 is half the number of terms, and 11 is the sum of the first and last terms. This suggests that the sum of a linear series is, $\sum_{1}^{n} i = \frac{n}{2}(1 + n) = \frac{n(n + 1)}{2}$
***Quadratic Series*:**	Similar methods help us to find the sum of a quadratic sequence, $\sum_{1}^{n} i^2 = \frac{n(n + 1)(2n + 1)}{6}$

***Summation Formulas*:**	***Constant Series*** $$\sum_{i=1}^{n} a_i = \sum_{i=1}^{n} A = nA$$ ***Linear Series*** $$\sum_{1}^{n} i = \frac{n(n+1)}{2}$$ ***Quadratic Series*** $$\sum_{1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$
***Example 3*:**	Evaluate each series, a. $$\sum_{i=5}^{10} 8$$ Use the summation formula. Notice there are 6 terms; therefore, $$\sum_{i=5}^{10} 8 = 6(8) = 48$$ b. $$\sum_{i=1}^{5} i$$ Use the summation formula. Notice there are 5 terms, and the sum of the first and last terms is 6. Therefore, $$\sum_{i=1}^{5} i = \frac{5(5+1)}{2} = \frac{5(6)}{2} = 15$$

c.

$$\sum_{i=1}^{7} i^2$$

Use the summation formula. Notice there are 7 terms; therefore,

$$\sum_{i=1}^{7} i^2 = \frac{7(7+1)(2\cdot 7+1)}{6} = \frac{7\cdot 8\cdot 15}{6} = 140$$

Exercise 3:

Evaluate each series,

a.

$$\sum_{i=1}^{60} 4$$

b.

$$\sum_{i=1}^{15} i$$

c.

$$\sum_{i=1}^{10} i^2$$

***Example 4*:**

Ricky is building a house of cards. He wants the house to have as many stories as possible. With a deck of 52 cards, how many stories will Ricky's house have?

1. **Understand the Problem**

   The answer will be the number of rows in the card house. List the important information:

   - He has 52 cards.
   - The house should have as many stories as possible.

2. **Make a Plan**

   - Make a diagram of the house.
   - Find a pattern for the number of cards in each story.
   - Write a series and evaluate it.

3. **Solve**

   Make a table and a diagram.

Row	1	2	3	4
Diagram	/\	/▽\	/▽▽\	/▽▽▽\
Cards	2	5	8	11

The number of cards increases by 3 in each row. Write a series to represent the total number of cards in *n* rows.

$$\sum_{i=1}^{n}(3i-1)$$

It is important to recognize that the sigma notation is addition. Therefore, we can use the associative and distributive properties to rewrite this expression as follows:

$$\sum_{i=1}^{n}(3i-1) = 3\sum_{i=1}^{n} i - \sum_{i=1}^{n} 1$$

The first sum is a linear series, and the second sum is a constant series. Therefore,

$$\sum_{i=1}^{n}(3i-1) = 3\frac{n(n+1)}{2} - n\cdot 1 = \frac{3n(n+1)}{2} - \frac{2n}{2}$$

$$= \frac{3n^2+3n-2n}{2}$$

$$= \frac{3n^2+n}{2}$$

Now, we set this equal to the total number of cards available (52) and solve for *n*.

$$\frac{3n^2+n}{2} = 52$$

$$3n^2+n-104 = 0$$

Using the quadratic formula, we find,

$$n \cong -6.06, 5.72$$

We reject the negative answer and recognize that we cannot have a fractional answer. Therefore, we cannot have more than 5 stories. We evaluate the series with $n = 5$.

$$\frac{3\cdot 5^2+5}{2} = 40$$

***Exercise 4*:**	A flexible garden hose is coiled for storage. Each subsequent loop is 6 inches longer than the preceding loop, and the innermost loop is 35 inches long. If there are 6 loops, how long is the hose?