Transition to College Math Name ________________________________

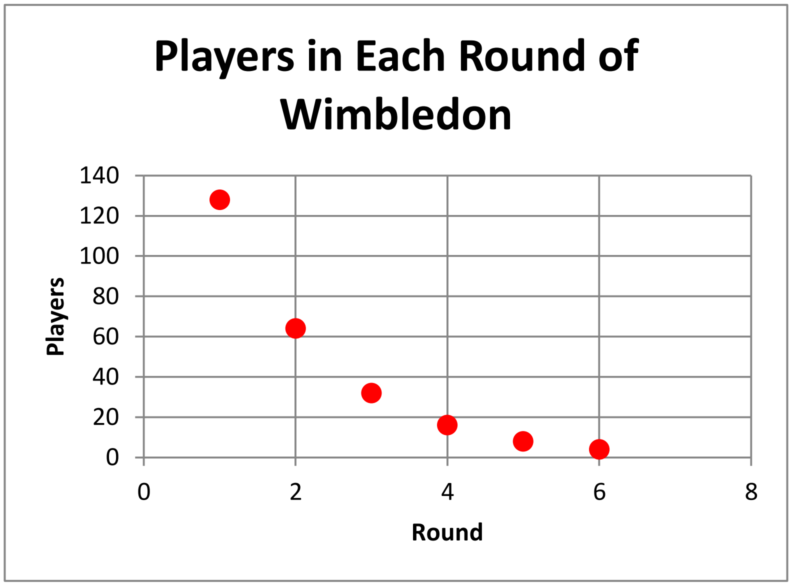
Period __________

Date: Unit: 1: **Series and Sequences** Lesson: **3: Geometric Sequences**	Essential Question: **The average of two numbers is also called their arithmetic mean. How is a geometric mean like an arithmetic mean? How is it different? What is the geometric mean of 12 and 108?**
Standard: F.LE.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
Learning Target:	80% of the students will be able to find the explicit and recursive rules for the following geometric sequence: 8, 12, 18, 27, …
Geometric Sequence:	Serena Williams was the winner out of 128 players who began the 2003 Wimbledon Ladies' Singles Championship. After each match, the winner continues to the next round, and the loser is eliminated from the tournament. This means that after each round only half of the players remain. The number of players remaining after each round can be modeled by a geometric sequence. In a ***geometric sequence***, the ratio of successive terms is a constant called the common ratio r $(r \neq 1)$. For the players remaining, r is ½.

Term	a_1	a_2	a_3	a_4
Value	128	64	32	16

Recall that exponential functions have a common ratio. When you graph the ordered pairs (n, a_n) of a geometric sequence, the points lie on an exponential curve as shown in the graph on the next page. Thus you can think of a geometric sequence as an exponential function with natural numbers as the domain.

Summary



Identifying Geometric Sequences:

In a geometric sequence, the ratio of successive terms must be the same for all pairs of successive terms. The following example illustrates how to use this fact to identify geometric sequences.

Example 1:

Determine whether each sequence could be geometric or arithmetic. If possible, find the common ratio or difference.

a. $8, 12, 18, 27, \ldots$

Difference: $4, 6, 9$

Ratio $\frac{3}{2}, \frac{3}{2}, \frac{3}{2}$

It could be geometric with $r = \frac{3}{2}$

b. $8, 16, 24, 32, \ldots$

Difference: $8, 8, 8$

Ratio $2, \frac{3}{2}, \frac{4}{3}$

It could be arithmetic with $d = 8$

c. $6, 10, 15, 21, \ldots$

Difference: $4, 5, 6$

Ratio $\frac{5}{3}, \frac{3}{2}, \frac{7}{5}$

It is neither.

***Exercise 1*:**	Determine whether each sequence could be geometric or arithmetic. If possible, find the common ratio or difference. a. $\frac{1}{4}, \frac{1}{12}, \frac{1}{36}, \frac{1}{108}, \ldots$ b. $1.7, 1.3, 0.9, 0.5, \ldots$ c. $-50, -32, -18, -8, \ldots$
***Recursive Rule*:**	Each term in a geometric sequence is the product of the previous term and the common ratio, giving the recursive rule for a geometric sequence. ***Recursive Rule*:** $$a_n = a_{n-1}r$$ Where r is the common ratio.
***Explicit Rule*:**	In addition to a recursive rule, geometric sequences also have an explicit rule. Each term is the product of the first term and a power of the common ratio as shown in the following table.

Round	1	2	3	n
Players	128	64	32	a_n
Formula	$128\left(\frac{1}{2}\right)^0$	$128\left(\frac{1}{2}\right)^1$	$128\left(\frac{1}{2}\right)^2$	$128\left(\frac{1}{2}\right)^{n-1}$

	This pattern can be generalized into an explicit rule for all geometric sequences. ***Explicit Rule*:** $$a_n = a_1 r^{n-1}$$ Where a_1 is the first term, and r is the common ratio. The following example illustrates the process for finding the n^th^ term of a given geometric sequence,
***Example 2*:**	Find the 9^{th} term of the geometric sequence $-5, 10, -20, 40, -80, \ldots.$ 1. Find the common ratio $r = \frac{a_2}{a_1} = \frac{10}{-5} = -2$ 2. Write a rule and evaluate for $n = 9$ $a_n = a_1 r^{n-1}$ $a_9 = -5(-2)^{9-1}$ $a_9 = -5(256) = -1280$ Check: Extend the sequence $a_5 = -80$ $a_6 = -80(-2) = 160$ $a_7 = 160(-2) = -320$ $a_8 = -320(-2) = 640$ $a_9 = 640(-2) = -1280$

Exercise 2:	Find the 9th term of each geometric sequence. a. $\frac{3}{4}, -\frac{3}{8}, \frac{3}{16}, -\frac{3}{32}, \ldots$ b. $0.001, 0.01, 0.1, 1, 10, \ldots$
Finding the p^th^ Term Given Two Terms:	Use the two terms to find the common ratio, *r*. Suppose the two terms we have are a_m and a_n. Then, $a_m = a_1 r^{m-1}$ $a_n = a_1 r^{n-1}$ First, we divide the left-hand sides and the right-hand sides of the above equations. $\frac{a_m}{a_n} = \frac{a_1 r^{m-1}}{a_1 r^{n-1}} = \frac{r^{m-1}}{r^{n-1}} = r^{(m-1)-(n-1)} = r^{m-n}$ Therefore, $r = \sqrt[(m-n)]{\frac{a_m}{a_n}}$

	Notice that if $m - n$ is even, you must consider both positive and negative values of the root. Transforming the first two equations above, $a_1 = a_m r^{1-m}$ $a_1 = a_n r^{1-n}$ Finally, if the missing term is the p^{th} term, then $a_p = a_1 r^{p-1}$
***Example 3*:**	Find the 10th term of the geometric sequence with $a_5 = 96$ and $a_7 = 384$. 1. Find the common ratio. $r = \sqrt[(7-5)]{\frac{384}{96}}$ $r = \pm 2$ 2. Find a_1. $a_1 = 96 \cdot 2^{1-5} = \frac{96}{16} = 6$ $a_1 = 384 \cdot 2^{1-7} = \frac{384}{64} = 6$ Because both 5 and 7 are odd, we would get the same results if we used $r = -2$. 3. Write the rule and evaluate for a_{10} First, consider the case $r = 2$. $a_{10} = 6(2)^{10-1} = 3072$ Then, consider the case $r = -2$. $a_{10} = 6(-2)^{10-1} = -3072$

	Therefore, the 10th term is either 3027 or –3027.
Exercise 3:	Find the 7th term of the geometric sequence with the given terms. a. $a_4 = -8$ and $a_5 = -40$ b. $a_2 = 768$ and $a_4 = 48$
Geometric Mean:	Geometric means are the terms between any two nonconsecutive terms of a geometric sequence. ***Geometric Mean:*** If a and b are positive terms of a geometric sequence with exactly one term between them, the geometric mean is given by the following expression: $\sqrt{ab}$

***Example 4*:**	Find the geometric mean of $\frac{1}{2}$ and $\frac{1}{32}$. $$\sqrt{ab} = \sqrt{\left(\frac{1}{2}\right)\left(\frac{1}{32}\right)} = \sqrt{\frac{1}{64}} = \frac{1}{8}$$
***Exercise 4*:**	Find the geometric mean of 16 and 25.
Class work:	p 659: 1-13
Homework:	p 659: 19-31, 51