Name ____

Period ___________

Date:

Unit: **1: Series and Sequences**

Lesson: **2: Arithmetic Sequences**

Essential Question: **The cells in a bee honeycomb are arranged in concentric hexagons about a central cell. The first hexagon has six cells. The second hexagon has 12 cells. The third hexagon has 18 cells. How many cells does the 10th hexagon have?**

Standard: F.LE.2

Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Learning Target:

Learn to identify arithmetic sequences and find the indicated terms of an arithmetic sequence by using an explicit rule and a recursive rule. 80% of the students will be able to answer the Essential Question.

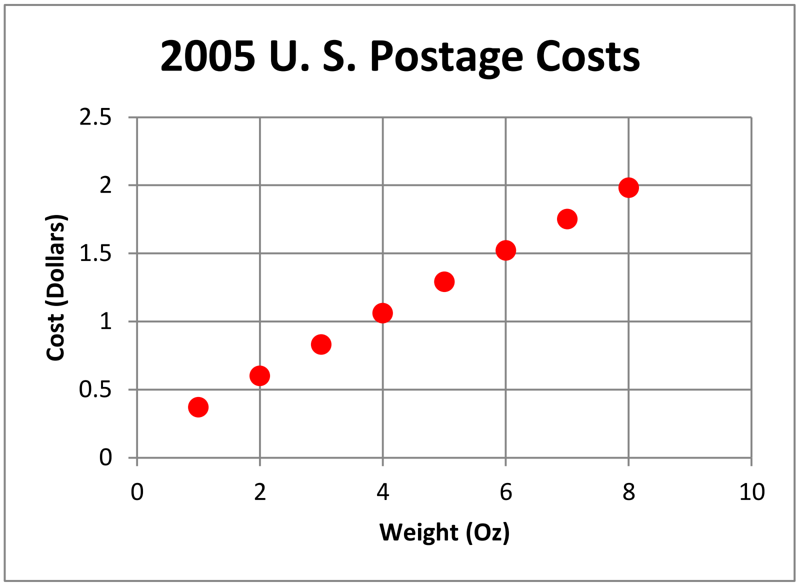
Arithmetic sequences:

You can use arithmetic sequences to predict the cost of mailing letters. The cost of mailing a letter in 2005 based on its weight in ounces gives the sequence 0.37, 0.60, 0.83, 1.06, …. This sequence is called an arithmetic sequence, because its successive terms differ by the same number d ($d \neq 0$), called the common difference. For the mail costs, d is 0.23, as shown.

n	1	2	3	4	...
a_n	0.37	0.60	0.83	1.06	...

Recall that linear functions have a constant first difference. Notice also that when you graph the ordered pairs (n, a_n) of an arithmetic sequence, the points lie on a straight line. Thus, you can think of an arithmetic sequence as a linear function with sequential natural numbers as the domain.

Summary



***Identifying Arithmetic Sequences*:**

A sequence is an arithmetic sequence if and only if the difference between the values of successive terms is the same for all pairs of terms.

***Definition of an Arithmetic Sequence*:**

If a_n and a_{n-1} are members of a sequence, S, then

$$\forall n > 1\colon\ a_n - a_{n-1} = d \ \Leftrightarrow\ S \text{ is an arithmetic sequence.}$$

Where d is the ***common difference***.

***Example 1*:**

Determine whether each sequence could be arithmetic. If so, find the common first difference and the next term.

a. 3, 2, 7, 12, 17, …
Differences 5 5 5 5
The sequence could be arithmetic with a common difference of 5.
The next term is 17+5=22.

b. –4, –12, –24, –40, –60, …
Differences –8 –12 –16 –20
The sequence is not arithmetic, because the first differences are not common.

Exercise 1:

Determine whether each sequence could be arithmetic. If so, find the common first difference and the next term.

a. 1.9, 1.2, 0.5, –0.2, –0.9, ...

b. $\frac{11}{2}, \frac{11}{3}, \frac{11}{4}, \frac{11}{5}, \frac{11}{6}, \cdots$

Recursive Rule:

Each term in an arithmetic sequence is the sum of the previous term and the common difference. This gives the recursive rule $a_n = a_{n-1} + d$. You also can develop an explicit rule for an arithmetic sequence. This table of the postage example above clearly depicts a pattern. Each term is the sum of the first term plus a multiple of the common difference.

Explicit Rule:

We can generalize this pattern into a rule for all arithmetic sequences.

> ***Explicit Rule:***
>
> The nth term of an arithmetic sequence is given by
>
> $$a_n = a_1 + (n - 1)d$$
>
> Where a_1 is the first term, and d is the common difference.

We often call an explicit rule a ***formula***.

You can use this formula to find the nth term of an arithmetic sequence. Example 3 illustrates this process.

***Example 2*:**

Find the 10th term of the arithmetic sequence,

$32, 25, 18, 11, 4, \ldots.$

1. Find the common difference.

   $d = 25 - 32 = -7$

2. Evaluate by using the formula.

   $a_n = a_1 + (n-1)d$

   $a_{10} = 32 + (10-1)(-7) = -31$

You can easily check this by continuing the sequence.

n	1	2	3	4	5	6	7	8	9	10
a_n	32	25	18	11	4	–3	–10	–17	–24	–31

***Exercise 2*:**

Find the 11th term of each arithmetic sequence. Then make a table for the first 11 terms of the sequence.

a. –3, –5, –7, –9, …

n	1	2	3	4	5	6	7	8	9	10	11
a_n											

b. 9.2, 9.15, 9.1, 9.05, …

n	1	2	3	4	5	6	7	8	9	10	11
a_n											

Finding Missing Terms:	Use the explicit rule to find the common difference. Then substitute the value for *n* into the explicit rule to find a_n.
Example 3:	Find the missing terms in the following arithmetic sequence: 11, ___, ___, ___, -17, …. Find *d* using $a_1 = 11$ and $a_5 = -17$. $a_n = a_1 + (n-1)d$ $-17 = 11 + (5-1)d$ $d = -7$ Find the missing terms using $d = -7$ and $a_1 = 11$. $a_2 = 11 + (2-1)(-7) = 4$ $a_3 = 11 + (3-1)(-7) = -3$ $a_4 = 11 + (4-1)(-7) = -10$
Exercise 3:	Find the missing terms in the following arithmetic sequence: 2, ___, ___, ___, 0, ….

Finding the nth term without finding the first term:

Suppose a_n and a_m are the nth and mth terms in an arithmetic sequence. The explicit rule gives a formula for finding the nth term of an arithmetic sequence.

$$a_n = a_1 + (n-1)d \tag{1}$$

Moreover, the explicit rule gives a formula for finding the mth term of an arithmetic sequence.

$$a_m = a_1 + (m-1)d \tag{2}$$

Solving equation (2) for a_1 gives,

$$a_1 = a_m - (m-1)d \tag{3}$$

Substituting the right hand side of equation (3) for a_1 in equation (1) gives,

$$a_n = a_m - (m-1)d + (n-1)d \tag{4}$$

Simplifying equation (4),

$$a_n = a_m + (n-m)d \tag{5}$$

Solving equation (5) for d

$$d = \frac{a_n - a_m}{n - m}$$

Therefore, you can use any two terms to find the common difference of an arithmetic sequence. Furthermore, you can use any two terms to find missing terms.

The example on the next page illustrates this process.

Example 4:

Find the 6th term of the arithmetic sequence with $a_9 = 120$ and $a_{14} = 195$.

$$d = \frac{a_n - a_m}{n - m} = \frac{195 - 120}{14 - 9} = \frac{75}{5}$$

$$d = 15$$

$$a_6 = a_9 + (6 - 9)d$$

$$a_6 = 120 + (-3)15 = 75$$

The 6th term is 75.

Exercise 4:

Find the 11th term of each arithmetic sequence.

a. $a_2 = -133$ and $a_3 = -121$

b. $a_3 = 20.5$ and $a_8 = 13$