Name ______________________________

Period ________

Date: Unit: **1: Series and Sequences** Lesson: **1: Sequences**	Essential Question: **Sequences are related to sets, but they are not the same. Consider the following sequence:** **0, 1, 0, 1, 0, 1, 0, 1, …** **What are the sets of the domain and range of this sequence?**
Standard: F.IF.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.*
***Learning Target*:** ***Sequence*:**	80% of the students will be able to find the 20th term of the sequence 6, 9, 12, 15,…, and 80% of the students will be able to write the rule for this sequence. A ***sequence*** is a ***function*** whose domain is the natural numbers, or a subset of the natural numbers. Its range is another set of numbers. We often represent the values of the sequence by using a subscript. The functional notation is, $a(n) = a_n$. We often write a sequence as an ordered list: $a_1, a_2, a_3, a_4, \ldots$ A sequence can have an infinite number of terms. We would use the above notation to represent an infinite sequence. If a sequence has a finite number of terms, say n terms, we would indicate that by writing the sequence like this, $a_1, a_2, a_3, \ldots, a_n$ It is important to recognize that there can be no gaps in the domain of a sequence. The number 1 is always a member of the domain of a sequence; moreover, if n is a member of the domain of a sequence and $n > 1$ then $n - 1$ is a member of the domain. After all, we are just counting the terms.
Summary	

***Terms*:**	The successive values of a sequence are called ***terms***. Each term has a term number (the domain) and a term value (the range.)
***Example 1*:**	The set of even numbers is an infinite sequence. It can be represented by using a table:

term number	n	1	2	3	4	5	...
term value	a_n	2	4	6	8	10	...

An example of a finite sequence might be all the integers less than 20 that are perfect squares

term number	n	1	2	3	4
term value	a_n	1	4	9	16

***Exercise 1*:**	Write an infinite sequence and a finite sequence that has at least four terms.

***Rule*:**	Every sequence must be defined by a ***rule***. This rule (or formula) determines the value of each term in the sequence. For example, the Fibonacci (Leonardo Fibonacci, c. 1170 – c. 1250) sequence can be represented by the following table:

term number	$\boldsymbol{n}$	1	2	3	4	5	...
term value	$\boldsymbol{a_n}$	1	1	2	3	5	...

***Recursive Rule*:**	The Fibonacci sequence is defined by a ***recursive*** rule. In a recursive sequence, the values of successive terms are determined from one or more of the previous terms.

Here is the rule for the Fibonacci sequence:

$$a_1 = 1,$$

$$a_2 = 1,$$

$$a_n = a_{n-1} + a_{n-2} \quad \Leftarrow \quad n > 2.$$

***Example 2*:**	Find the first five terms of the sequence that has the following recursive rule:

$$a_1 = 3,$$

$$a_n = 3a_{n-1} - 8 \quad \Leftarrow \quad n > 1.$$

term number	$\boldsymbol{n}$	1	2	3	4	5	...
term value	$\boldsymbol{a_n}$	3	1	–5	–23	–77	...

Notice that the term value decreases as the term number increases.

Exercise 2:

Find the first five terms of the sequence that has the following recursive rule:

$$a_1 = 3,$$
$$a_n = 2a_{n-1} + 3 \;\Leftarrow\; n > 1.$$

Term number	***n***	1	2	3	4	5	...
term value	a_n						...

Formula Rule:

Some sequences are defined by a ***formula***. If you know the formula that defines a sequence, you do not need to know any of the preceding terms in order to determine the n^{th} term.

Example 3:

Find the first five terms of the sequence that has the following formula:

$$a_n = n^2 - 5 \;.$$

term number	***n***	1	2	3	4	5	...
term value	a_n	–4	–1	4	11	20	...

Exercise 3:

Find the first five terms of the sequence that has the following formula:

$$a_n = \tfrac{1}{2}(n+1)(n+2) \;.$$

term number	***n***	1	2	3	4	5	...
term value	a_n						...

***Finding the Rule*:** Discovering the rule for a sequence requires you to think logically and to apply your knowledge of algebra particularly functions. Often a good place to start is to the take the differences between successive terms.

***Example 4*:** Find the rule for the following sequence:

$\boldsymbol{n}$	1	2	3	4	5	…	n
$\boldsymbol{a_n}$	1	4	7	10	13	…	$3n - 2$
$\boldsymbol{a_n - a_{n-1}}$		3	3	3	3		

If the first difference is constant, then the rule will be linear with the difference multiplying n. If m is the difference between terms, then the rule will have this form,

$$mn + b.$$

You can find m by setting this rule equal to the value of any term and solving for b. In this case, let's choose $n = 4$:

$$a_4 = 3 \cdot 4 + b.$$

$$10 = 12 + b.$$

$$b = -2.$$

***Exercise 4*:** Find the rule for the following sequence:

$\boldsymbol{n}$	1	2	3	4	5	…	n
$\boldsymbol{a_n}$	1	−1	−3	−5	−7	…	
$\boldsymbol{a_n - a_{n-1}}$							

***Example 5*:**	Find the rule for the following sequence:

n	1	2	3	4	5	…	n
$\boldsymbol{a_n}$	2	4	8	16	32	…	$3n-2$
first difference		2	4	8	16		
second difference			2	4	8		

Obviously, the differences will never be constant. However, the ***ratios*** are constant.

$$a_n = 2a_{n-1} \quad \Leftarrow \quad n > 1.$$

In this case, the ratio is 2. Look for a pattern that includes powers of 2.

$$a_n = m2^{n+b}.$$

If we look at the first term, we see that

$a_n = 2^n$ is a correct rule.

Notice that $a_n = 2 \cdot 2^{n-1}$ also describes the sequence correctly. It is possible to find more than one rule for the n^{th} term of a sequence.

Exercise 5:	Find the rule for the following sequence:

n	1	2	3	4	5	…	n
a_n	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	…	