Name ___________________________________

Period __________

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| Date:<br><br>Unit: **1: Series and Sequences**<br><br>Lesson: **1: Sequences** | Essential Question: **Sequences are related to sets, but they are not the same. Consider the following sequence:**<br>**0, 1, 0, 1, 0, 1, 0, 1, …**<br>**What are the sets of the domain and range of this sequence?** |
| Standard: F.IF.3 | Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.* |
| ***Learning Target*:**<br><br><br><br>***Sequence*:** | 80% of the students will be able to find the 20th term of the sequence 6, 9, 12, 15,…, and 80% of the students will be able to write the rule for this sequence.<br><br>A ***sequence*** is a ***function*** whose domain is the natural numbers, or a subset of the natural numbers. Its range is another set of numbers. We often represent the values of the sequence by using a subscript. The functional notation is,<br>$a(n) = a_n$.<br>We often write a sequence as an ordered list:<br>$a_1, a_2, a_3, a_4, \ldots$<br>A sequence can have an infinite number of terms. We would use the above notation to represent an infinite sequence.<br>If a sequence has a finite number of terms, say n terms, we would indicate that by writing the sequence like this,<br>$a_1, a_2, a_3, \ldots, a_n$<br>It is important to recognize that there can be no gaps in the domain of a sequence. The number 1 is always a member of the domain of a sequence; moreover, if $n$ is a member of the domain of a sequence and $n > 1$ then $n - 1$ is a member of the domain. After all, we are just counting the terms. |
| Summary | |

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| ***Terms*:** | The successive values of a sequence are called ***terms***. Each term has a term number (the domain) and a term value (the range.) |
| ***Example 1*:** | The set of even numbers is an infinite sequence. It can be represented by using a table: |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| ***term number*** | $n$ | 1 | 2 | 3 | 4 | 5 | ... |
| ***term value*** | $a_n$ | 2 | 4 | 6 | 8 | 10 | ... |

An example of a finite sequence might be all the integers less than 20 that are perfect squares

| | | | | | |
|---|---|---|---|---|---|
| ***term number*** | $n$ | 1 | 2 | 3 | 4 |
| ***term value*** | $a_n$ | 1 | 4 | 9 | 16 |

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| ***Exercise 1*:** | Write an infinite sequence and a finite sequence that has at least four terms. |

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| ***Rule*:** | Every sequence must be defined by a ***rule***. This rule (or formula) determines the value of each term in the sequence. For example, the Fibonacci (Leonardo Fibonacci, c. 1170 – c. 1250) sequence can be represented by the following table: |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| ***term number*** | ***n*** | 1 | 2 | 3 | 4 | 5 | ... |
| ***term value*** | $a_n$ | 1 | 1 | 2 | 3 | 5 | ... |

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| ***Recursive Rule*:** | The Fibonacci sequence is defined by a ***recursive*** rule. In a recursive sequence, the values of successive terms are determined from one or more of the previous terms. |

Here is the rule for the Fibonacci sequence:

$$a_1 = 1,$$

$$a_2 = 1,$$

$$a_n = a_{n-1} + a_{n-2} \Leftarrow n > 2.$$

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| ***Example 2*:** | Find the first five terms of the sequence that has the following recursive rule: |

$$a_1 = 3,$$

$$a_n = 3a_{n-1} - 8 \Leftarrow n > 1.$$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| ***term number*** | ***n*** | 1 | 2 | 3 | 4 | 5 | ... |
| ***term value*** | $a_n$ | 3 | 1 | −5 | −23 | −77 | ... |

Notice that the term value decreases as the term number increases.

***Exercise 2*:**

Find the first five terms of the sequence that has the following recursive rule:

$$a_1 = 3,$$
$$a_n = 2a_{n-1} + 3 \Leftarrow n > 1.$$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| ***Term number*** | ***n*** | 1 | 2 | 3 | 4 | 5 | ... |
| ***term value*** | $\boldsymbol{a_n}$ | | | | | | ... |

***Formula Rule*:**

Some sequences are defined by a ***formula***. If you know the formula that defines a sequence, you do not need to know any of the preceding terms in order to determine the $n^{\text{th}}$ term.

***Example 3*:**

Find the first five terms of the sequence that has the following formula:

$$a_n = n^2 - 5 \ .$$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| ***term number*** | ***n*** | 1 | 2 | 3 | 4 | 5 | ... |
| ***term value*** | $\boldsymbol{a_n}$ | $-4$ | $-1$ | 4 | 11 | 20 | ... |

***Exercise 3*:**

Find the first five terms of the sequence that has the following formula:

$$a_n = \frac{1}{2}(n+1)(n+2) \ .$$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| ***term number*** | ***n*** | 1 | 2 | 3 | 4 | 5 | ... |
| ***term value*** | $\boldsymbol{a_n}$ | | | | | | ... |

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| ***Finding the Rule*:** | Discovering the rule for a sequence requires you to think logically and to apply your knowledge of algebra particularly functions. Often a good place to start is to the take the differences between successive terms. |
| ***Example 4*:** | Find the rule for the following sequence: |

| $n$ | 1 | 2 | 3 | 4 | 5 | … | $n$ |
|---|---|---|---|---|---|---|---|
| $a_n$ | 1 | 4 | 7 | 10 | 13 | … | $3n-2$ |
| $a_n - a_{n-1}$ | | 3 | 3 | 3 | 3 | | |

If the first difference is constant, then the rule will be linear with the difference multiplying $n$. If $m$ is the difference between terms, then the rule will have this form,

$$mn + b.$$

You can find m by setting this rule equal to the value of any term and solving for $b$. In this case, let's choose $n = 4$:

$$a_4 = 3 \cdot 4 + b.$$

$$10 = 12 + b.$$

$$b = -2.$$

***Exercise 4*:** Find the rule for the following sequence:

| $n$ | 1 | 2 | 3 | 4 | 5 | … | $n$ |
|---|---|---|---|---|---|---|---|
| $a_n$ | 1 | –1 | –3 | –5 | –7 | … | |
| $a_n - a_{n-1}$ | | | | | | | |

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| ***Example 5:*** | Find the rule for the following sequence: |

| **n** | 1 | 2 | 3 | 4 | 5 | … | $n$ |
|---|---|---|---|---|---|---|---|
| **$a_n$** | 2 | 4 | 8 | 16 | 32 | … | $3n-2$ |
| **first difference** | | 2 | 4 | 8 | 16 | | |
| **second difference** | | | 2 | 4 | 8 | | |

Obviously, the differences will never be constant. However, the ***ratios*** are constant.

$$a_n = 2a_{n-1} \Leftarrow n > 1.$$

In this case, the ratio is $2$. Look for a pattern that includes powers of $2$.

$$a_n = m2^{n+b}.$$

If we look at the first term, we see that

$a_n = 2^n$ is a correct rule.

Notice that $a_n = 2 \cdot 2^{n-1}$ also describes the sequence correctly. It is possible to find more than one rule for the $n^{\text{th}}$ term of a sequence.

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| ***Exercise 5*:** | Find the rule for the following sequence: |

| $n$ | 1 | 2 | 3 | 4 | 5 | … | $n$ |
|---|---|---|---|---|---|---|---|
| $a_n$ | 1 | $\frac{1}{2}$ | $\frac{1}{3}$ | $\frac{1}{4}$ | $\frac{1}{5}$ | … | |

**Class work:** p 629: 1-15

**Homework:** p 629 ff: 16-58 even