

Chapter 1.4

Parametric Equations

Objectives

- graph curves that are described using parametric equations
- find parameterizations of circles, ellipses, line segments, and other curves

Learning Target

80% of the students will be able to write the equation of a circle centered on the origin in the x-y plane in terms of a parameter t.

Standard

G-GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

Overview

- Relations
- Circles
- Ellipses
- Lines and other Curves

Relation

- Set of ordered pairs (x, y) of real numbers
- Graph is a corresponding set of points in a plane
- If x and y are functions of a third variable t
 - Parameter
 - Parametric mode

Example 1 Graphing Half a Parabola

- Describe the graph determined by
 - $x = \sqrt{t}$
 - $y = t$
 - $t \geq 0$
- Indicate the direction in which the curve is traced.
- Find a Cartesian equation for the curve that contains the solution.

Example 1 Solution

- Set the parametric mode of the grapher.
- Set
 - $x = \sqrt{t}$
 - $y = t$
- Set window

```

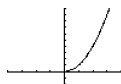
          SCI  ENG
  MODE  V I 2 3 4 5 6 7 8 9
  F1(F2) DEGREE
  F3(F4) POL  SEQ
  F5(F6) PBL  SET
  F7(F8) DRT  SEQ
  F9(F10) SIMUL
  F11(F12) P-△
  F13(F14) G-T
  F15(F16) MENT+

  P101 P102 P103
  X1T 0.1
  Y1T 0.1
  X2T =
  Y2T =
  X3T =
  Y3T =

  WINDOW
  Tmin=
  Tmax=25
  Tstep=.1308996...
  Xmin=-4
  Xmax=4
  Xsc1=1
  Ymin=-2
  
```

Example 1 Solution Continued

- Display graph
- Right half of $y = x^2$
- No information about t
- Appears to start at $(0, 0)$



Example 1 Confirm Algebraically

- Because $t \geq 0$ both
 - $x \geq 0$
 - $y \geq 0$
- Eliminating t
 - $y = t = (\sqrt{t})^2 = x^2$
- $\therefore y = x^2, \quad x \geq 0$

Exercise 1

- Graph
 - $x = -\sqrt{t}$
 - $y = 9t^2$
 - $t \geq 0$
- Indicate direction in which curve is traced
- Find Cartesian equation

Parametric Curve, Equations

- If x and y are given as functions,
 - $x = f(t)$
 - $y = g(t)$
 - t -interval
- $(f(t), g(t)),$ parametric curve
- $x = f(t), y = g(t)$ parametric equations

Parametric Curve

- **Parameter** is t
- Domain of t , I , is **parametric interval**
- If I is closed interval, $a \leq t \leq b$
 - $(f(a), g(a))$ is **initial point**
 - $(f(b), g(b))$ is **terminal point**
- Parametric equations
 - We have **parameterized** the curve
 - Equations and interval: **parameterization**

What does t mean?

- Time
- Angle
- Distance traveled
- Simulate motion

Example 2 Graphing a Circle

Describe the graph of the relation determined by

$$x = 2 \cos t \quad y = 2 \sin t \quad 0 \leq t \leq 2\pi$$

Find the initial and terminal points, if any, and the direction in which the curve is traced.

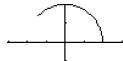
Find a Cartesian equation for a curve that contains the parametrized curve.

Example 2, continued

$$\bullet 0 \leq t \leq \pi/4$$



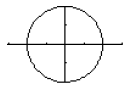
$$\bullet 0 \leq t \leq 3\pi/4$$



$$\bullet 0 \leq t \leq 7\pi/4$$



$$\bullet 0 \leq t \leq 2\pi$$



Example 2, continued

$$x = 2 \cos t \quad y = 2 \sin t$$

$$x^2 + y^2 = 4 \cos^2 t + 4 \sin^2 t$$

$$= 4(\cos^2 t + \sin^2 t) = 4$$

$$x^2 + y^2 = 4$$

Exercise 2

Describe the graph of the relation determined by

$$x = 3 \sin t \quad y = 3 \cos t \quad 0 \leq t \leq 2\pi$$

Find the initial and terminal points, if any, and the direction in which the curve is traced.

Find a Cartesian equation for a curve that contains the parametrized curve.

Parameterizing Circles

```

P1: Plot Plot2
V1: Rcos(T)
V2: Rsin(T)
V3:
V4:
V5:
V6:

```

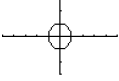
```

WINDOW
Tmin=0
Tmax=6.2831853...
Tstep=.1308996...
Xmin=-5
Xmax=5
Xsc1=1
Ymin=-3

```

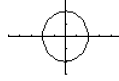
1→R

1



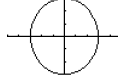
2→R

2



3→R

3



Direction?

$0 \leq t \leq \frac{\pi}{3}$

$0 \leq t \leq \pi$

$0 \leq t \leq \frac{4\pi}{3}$

```

WINDOW
Tmin=0
Tmax=1.0471975...
Tstep=.1308996...
Xmin=-5
Xmax=5
Xsc1=1
Ymin=-3

```

```

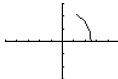
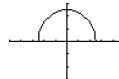
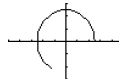
WINDOW
Tmin=0
Tmax=3.1415926...
Tstep=.1308996...
Xmin=-5
Xmax=5
Xsc1=1
Ymin=-3

```

```

WINDOW
Tmin=0
Tmax=4.1887902...
Tstep=.1308996...
Xmin=-5
Xmax=5
Xsc1=1
Ymin=-3

```

Example 3 Graphing an Ellipse

Describe the graph of the relation determined by

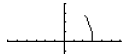
$$x = 3 \cos t \quad y = 4 \sin t \quad 0 \leq t \leq 2\pi$$

Find the initial and terminal points, if any, and the direction in which the curve is traced.

Find a Cartesian equation for a curve that contains the parametrized curve.

Example 3, continued

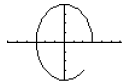
$$\bullet 0 \leq t \leq \pi/4$$



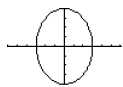
$$\bullet 0 \leq t \leq 3\pi/4$$



$$\bullet 0 \leq t \leq 7\pi/4$$



$$\bullet 0 \leq t \leq 2\pi$$



Example 2, continued

$$\frac{x}{3} = \cos t \quad \frac{y}{4} = \sin t$$

$$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{4}\right)^2 = \cos^2 t + \sin^2 t$$

$$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{4}\right)^2 = 1$$

Exercise 3

Describe the graph of the relation determined by

$$x = 4 \sin t \quad y = 3 \cos t \quad 0 \leq t \leq 2\pi$$

Find the initial and terminal points, if any, and the direction in which the curve is traced.

Find a Cartesian equation for a curve that contains the parametrized curve.

Ellipses

- Graph
 - $x = 5 \cos t$
 - $y = 2 \sin t$
 - $0 \leq t \leq 2\pi$

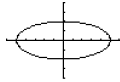
```

Plot1 Plot2 Plot3
V1t 5cos(T)
V2t 2sin(T)
V3t =
V4t =
V5t =
V6t =

WINDOW
Tmin=0
Tmax=6.2831853...
Tstep=.05
Xmin=-5
Xmax=5
Xscl=1
Ymin=-3

```

$$\left(\frac{x}{5}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$$



Parameterizing Ellipses

```

Plot1 Plot2 Plot3
V1t Bcos(T)
V2t Bsin(T)
V3t =
V4t =
V5t =
V6t =

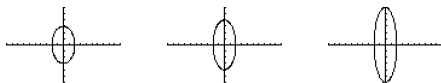
```

```

WINDOW
Tmin=0
Tmax=6.2831853...
Tstep=.05
Xmin=-10
Xmax=10
Xscl=1
Ymin=-6

```

2→B 2 4→B 4 6→B 6
 3→B 3



Example 4 Graphing a Line Segment

- Graph
 - $x = 3t$
 - $y = 2 - 2t$
 - $0 \leq t \leq 1$
- Line segment
 - (0, 2)
 - (3, 0)
- $y = -\frac{2}{3}x + 2$

```

Plot1 Plot2 Plot3
V1t 3T
V2t 2-2T
V3t =
V4t =
V5t =
V6t =

WINDOW
Tmin=0
Tmax=1
Tstep=.05
Xmin=-4
Xmax=4
Xscl=1
Ymin=-2

```



Exercise 4

- Graph
 - $x = 3 - 3t$
 - $y = 2t$
 - $0 \leq t \leq 1$
- Indicate direction in which curve is traced
- Find Cartesian equation
- What portion of the Cartesian equation is graphed?

Example 5

Parameterizing a Line Segment

- Find a parameterization for the line segments with endpoints $(-2, 1)$ and $(3, 5)$.
- Using the point $(-2, 1)$,
 - $x = -2 + at$
 - $y = 1 + bt$
 - $\frac{x+2}{a} = t = \frac{y-1}{b}$
- If $t = 0$, the point $(-2, 1)$ is a solution.

Example 5

Parameterizing a Line Segment

- Find a and b so that $x = 3$ and $y = 5$ when $t = 1$.
- $3 = -2 + a \Rightarrow a = 5$
- $5 = 1 + b \Rightarrow b = 4$
- $x = -2 + 5t \quad y = 1 + 4t \quad 0 \leq t \leq 1$

$\sqrt{1+16}$ Plot2 P1a13
 $\sqrt{1+16}$ $2+5t$
 $\sqrt{1+16}$ $1+4t$
 $\sqrt{2t} =$
 $\sqrt{3t} =$
 $\sqrt{4t} =$
 $\sqrt{5t} =$



Exercise 5

Find a parameterization for the line segments with endpoints $(-1, -3)$ and $(4, 1)$.

Witch of Agnesi

- Graph

- $x = 2 \cot t$
- $y = 2 \sin^2 t$
- $0 \leq t \leq \pi$

```

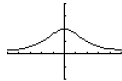
F1(t) F1(t) F1(t)
√X1=2/∫an(T)
√Y1=2sin(T)²
√Z1=
√Z1=
√Z1=
√Z1=

```

```

WINDOW
Xmin=0
Xmax=3.1415926...
Xstep=.05
Ymin=-5
Ymax=5
Xsc1=1
Ymin=-2

```



Homework

(p. 34): #1-4 all, 5-15 odds
