Chapter 1.3

Exponential Functions

Objectives

- determine the domain, range, and graph of an exponential function
- solve problems involving exponential growth and decay
- use exponential regression equations to solve problems

Learning Target

80% of the students will be able to compare the time it takes for an investment to double at an interest rate of 8%, compounded quarterly, and compounded continuously.

Standard

G-GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

Overview

- Exponential Growth
- Exponential Decay
- Applications
- The Number *e*

Exponential Growth

Suppose we invest $100 at 6% interest compounded annually.

After one year we have	$\$100 \cdot 1.06 = \$106$
After two years we have	$\$106 \cdot 1.06 = \$112.36$
After three years we have	$\$119.1016$
After four years we have	$\$126.247696$
After n years we have	$\$100 \cdot 1.06^n$

Exponential Growth

Exploration

Graph $y = a^x$ with $a = 2,3,5$

Graph $y = a^{-x}$ with $a = 2,3,5$

Exponential Function

Let a be a positive real number other than 1. the function

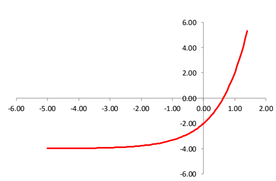
$$f(x) = a^x$$

is the ***exponential function with base a.***

Domain: $(-\infty, \infty)$
Range: $(0, \infty)$

Example 1

Graph the function
$y = 2(3^x) - 4$



Domain: $(-\infty, \infty)$

Range: $(-4, \infty)$

Exercise 1

Graph the function

$y = 3^{2x} - 1$

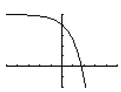
What are its domain and range

Example 2

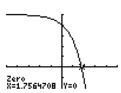
Find the zeros of

$f(x) = 5 - 2.5^{x}$

Graph



Zeros



Exercise 2

Find the zeros of

$f(x) = 0.6^{x} - 2$

Rules for Exponents

If $a > 0, b > 0$, and $x, y \in \mathbb{R}$, then

- $a^x \cdot a^y = a^{x+y}$
- $\frac{a^x}{a^y} = a^{x-y}$
- $(a^x)^y = (a^y)^x = a^{xy}$
- $a^x \cdot b^x = (ab)^x$
- $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$

Population Growth

US population

Year	Population	Ratio
1998	276.1	
1999	279.3	1.0116
2000	282.4	1.0111
2001	285.3	1.0102
2002	288.2	1.0102
2003	291.0	1.0097

Example 3

Use the data in the previous slide to predict the US population in 2010.

It looks like the population in the US increases by ~1% each year.

Using this conjecture and 1998 as the base year, the population in 2010 would be modeled by

$$276.1 \cdot 1.01^{12} \approx 311.1$$

This indicates the US population would be 311.1 million in 2010.

In fact, it was 309.4 million.

Exercise 3

This table gives the population of Nevada in millions.

Year	Population	Ratio
1998	1.853	
1999	1.935	
2000	1.998	
2001	2.095	
2002	2.165	
2003	2.241	

Use an exponential model to estimate the population in 2010.

Exponential Decay

Half life is the time for a radioactive substance to decay to another substance.

For example, a ^{8}Li nucleus decays to ^{8}Be by emitting an electron.

Half of the ^{8}Li nuclei in a sample will decay in 0.838 s.

Therefore, ^{8}Li has a half life of 0.838 s.

Example 4

Suppose a 5 g sample of a certain radioactive substance has a half life of 20 days. When will there be 1 g of the substance left?

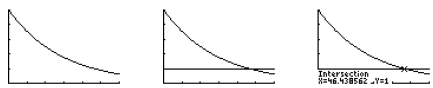
After 20 days: $5 \cdot \frac{1}{2}$

After 40 days: $5 \cdot \left(\frac{1}{2}\right)^2$

After t days: $5 \cdot \left(\frac{1}{2}\right)^{t/20}$

Example 4. continued

Solve graphically



There will be 1 g of the substance left after about 46.44 days.

Exercise 4

^{90}Sr has a half life of 28.8 years. How many years will 1 g of ^{90}Sr take to decay to 0.1 g?

Exponential Growth and Decay

If $k > 0$, then the function $y = k \cdot a^x$ is a model for,

exponential growth if $a > 1$

exponential decay if $0 < a < 1$

Exponential Regression

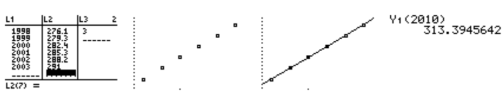
Most graphing calculators have the ability to fit an exponential model to a data set.

This is called an "***exponential regression equation***".

In fact, it is usually only an approximation to the best fit curve, but it is, usually, very close.

Example 5

Use the data in the US population table above to estimate the US population in 2010.



The actual population was 309.4 million

Exercise 5

Use this table and an exponential regression analysis to estimate the population of Texas in 2003.

Year	Population (millions)
1980	14.229
1990	16.986
1995	18.959
1998	20.158
1999	20.558
2000	20.852

Example 6

In Example 5, the base of the exponential was approximately $b = 1.0105$.

Therefore, the predicted annual rate of growth of the US population is $b - 1 = 0.0105$.

or about 1.05%

Exercise 6

Use your results from Exercise 5 to estimate the annual growth rate of the population of Texas.

The Number e

Many phenomena are best modeled by an exponential function that uses the number e as a base.

$$e \approx 2.718281828$$

If interest is compounded continuously, then the formula used would be,

$$y = Pe^{rt}$$

where P is the initial investment, r is the interest rate (expressed as a decimal) and t is the number of time periods.

Homework

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- 21-31odds,
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