# Chapter 1-2

Functions and Graphs

## Objectives

- Functions
- Domains and Ranges
- Viewing and Interpreting Graphs
- Even Functions and Odd Functions – Symmetry
- Functions Defined in Pieces
- Absolute Value Function
- Composite Functions

## Learning Target

- 80% of the students will be able to use a graphing calculator to graph The following functions: $f(x) = \frac{1}{x}$ and $f(x) = \sqrt{x}$.

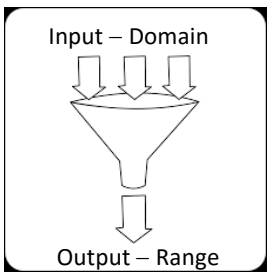
## Standard

F-BF.1c Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*

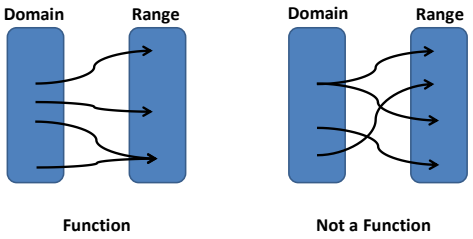
## Functions

- Functions relate one variable to another.
- $Circumference = \pi \times Diameter$
- $Diameter$ is the **independent** variable.
- $Circumference$ is the **dependent** variable.

## Functions



## Functions




A ***function*** is a correspondence between two sets: the domain, *D*, and the range, *R*, which assigns to each member of *D* to ***exactly*** one member of *R*.

## Function Notation

- $y = f(x)$ represents a function of $x$.
- $y = f(x)$ is read, "y equals f of x".
- Typically, it is set equal to an expression containing $x$ and no other variables.

## Example 1

Write a formula that expresses the area of a circle to its radius.

If the radius of the circle is r, then the area of the circle can be expressed as $A(r) = \pi r^2$.

Use the formula to find the area of a circle with radius 2 in.

$$A(2) = \pi(2)^2 = 4\pi$$

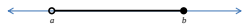
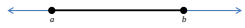
The area of a circle with radius $2\ in$ is $4\pi\ in^2$.

## Exercise 1

Write a formula that expresses the volume of a cube to its edge $e$.

Find the volume of a cube with edge 5 m.

## Interval Notation

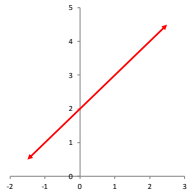
- $a < x < b$ $(a, b)$ 
- $a \leq x < b$ $[a, b)$ 
- $a < x \leq b$ $(a, b]$ 
- $a \leq x \leq b$ $[a, b]$ 

## Boundary Points

- The circles represent boundary points.
- The remaining points in the interval are interior points.
- Closed intervals contain their boundary points.
- Open intervals contain no boundary points.
  - Every point is an interior point.

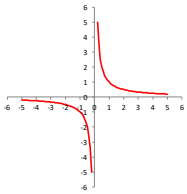
## Interpreting Graphs

- All the ordered pairs that satisfy a function make up that functions graph.
- $y = f(x) = x + 2$



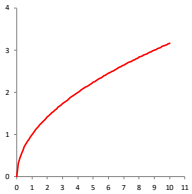
## Example 2 Inferring Domain and Range of a Function

- $y = \frac{1}{x}$
- Domain: $\{x \in \mathbb{R}: x \neq 0\}$
- Range: $\{y \in \mathbb{R}: y \neq 0\}$



## Example 2 Inferring Domain and Range of a Function, continued

- $y = \sqrt{x}$
- Domain: $\{x \in \mathbb{R}: x \geq 0\}$
- Range: $\{y \in \mathbb{R}: y \geq 0\}$



## Exercise 2

Find the domain and range of

$$f(x) = x^2 - 9$$

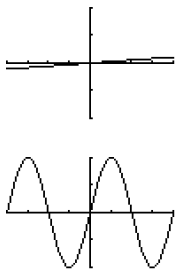
Sketch a graph
of the function.

## Graph Viewing Skills

1. Recognize that the graph is reasonable.
2. See all the important characteristics of the graph.
3. Interpret those characteristics.
4. Recognize grapher failure.

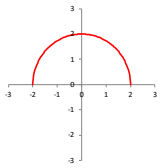
## Example of Grapher Failure

- $y = \sin x$



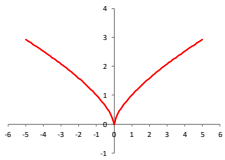
## Example 3
## Identifying Domain and Range

- $y = \sqrt{4 - x^2}$
- Domain: $\{x \in \mathbb{R}: -2 \leq x \leq 2\}$
- Range: $\{y \in \mathbb{R}: y \geq 0\}$



## Example 3
## Identifying Domain and Range, continued

- $y = x^{2/3}$
- Domain: $\mathbb{R}$
- Range: $\{y \in \mathbb{R}: y \geq 0\}$



## Exercise 3

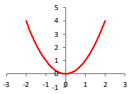
Use a grapher to find the domain and range of

$$y = \sqrt[3]{1 - x^2}$$

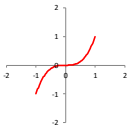
Sketch a graph
of the function.

## Example 4 Even and Odd Function

- A function is ***even*** if $f(-x) = f(x)$



- A function is ***odd*** if $f(-x) = -f(x)$



## Exercise 4

Without writing anything except the answer, determine whether the following functions are odd, even, or neither.

a. $y = x + x^2$

b. $y = x^2 - 3$

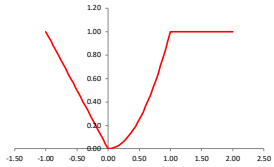
## Piecewise-Defined Functions

- Some functions are defined by one formula over their entire domain.
- Others are defined by applying different formulas on different parts of the domain.
- This type of function is called a piecewise-defined function.

## Example 5
## Graphing piecewise-defined functions

Graph

$$y = f(x) = \begin{cases} -x, & x < 0 \\ x^2, & 0 \le x \le 1 \\ 1, & x > 1 \end{cases}$$



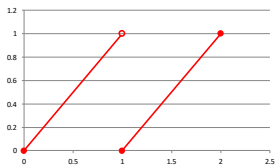

## Exercise 5

Graph

$$y = f(x) = \begin{cases} x^2, & x < 0 \\ x^3, & 0 \le x \le 1 \\ 2x - 1, & x > 1 \end{cases}$$

## Example 6
## Writing Formulas for Piecewise Functions

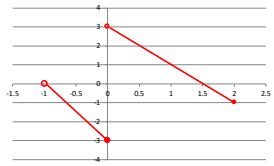
Write a function for the following graph:



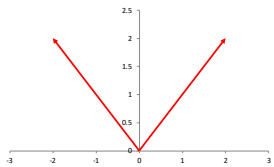

$0 \le x < 1 \Rightarrow y = x$ $\quad 1 \le x \le 2 \Rightarrow y = x - 1$ $\quad y = \begin{cases} x, 0 \le x < 1 \\ x - 1, 1 \le x \le 2 \end{cases}$

## Exercise 6

Write a piecewise function for the following graph:



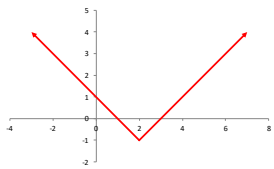
## Absolute Value Function



$$y = |x| = \begin{cases} -x, x < 0 \\ 0, x = 0 \\ x, x > 0 \end{cases}$$

## Example 7
## Transformations

Draw the graph of $f(x) = |x - 2| - 1$



Then find the domain and range.

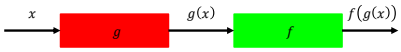
Domain: $(-\infty, \infty)$ Range: $[-1, \infty)$

## Exercise 7

Draw the graph of $f(x) = 2|x + 4| - 3$
Then find the domain and range.

## Composing Functions

Composing functions means using the output of one function as the input for another function.




$$f \circ g(x) = f\big(g(x)\big)$$

## Example 8
## Composing Functions

Find a formula for $f\big(g(x)\big)$
if $g(x) = x^2$
and $f(x) = x - 7$

Replace $x$ in $f(x)$ with $g(x)$.

$$f\big(g(x)\big) = g(x) - 7 = x^2 - 7$$

$$f\big(g(2)\big) = 2^2 - 7 = -3$$

## Exercise 8

Using $f(x)$ and $g(x)$ from Example 8, find

$$g(f(x))$$

Then find

$$g(f(2))$$

## Homework

Section 1.2 (p. 19): #1-4 all, 5-11 odds, 14-20 evens, 21-33 odds, 37-53 odds, 55 (do not expand expression), 56a