Chapter 1.1

Lines

Objectives

- use increments to calculate slopes
- write an equation and sketch a graph a line given specific information
- identify the relationships between parallel lines, perpendicular lines, and slopes
- use linear regression equations to solve problems

Learning Target

80% of the students will be able to find the equation of a line, given two points on the line.

Standard

G-GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

Overview

- Increments
- Slope of a Line
- Parallel and Perpendicular Lines
- Equations of Lines
- Applications

Calculus

- Calculus was invented to help physicists understand motion.
- Calculus relates rate of change of a quantity to a graph of the quantity.
- Explaining that relationship is the goal of this course.
- We will start by examining slopes.

Increments

- Particle in motion:
  - Changes in position are ***increments***.
  - Subtract the coordinates of its starting point from the coordinates of its ending point.

Definition

- If a particle moves from the point (x_1, y_1) to the point (x_2, y_2), the increments are,
  - $\Delta x = x_2 - x_1$ $\quad\quad$ $\Delta y = y_2 - y_1$
- The symbol, Δ, is the capital Greek letter equivalent to "D".
- Δx represents the difference between two values of x.
- Δx does ***not*** indicate multiplication.

Example 1: Finding Increments

- Find the coordinate increments if a particle moves from $(4, -3)$ to $(2,5)$.
  - $\Delta x = 2 - 4 = -2$
  - $\Delta y = 5 - (-3) = 8$
- Find the coordinate increments if a particle moves from $(5,6)$ to $(5,1)$.
  - $\Delta x = 5 - 5 = 0$
  - $\Delta y = 1 - 6 = -5$

Exercise 1

- Find the coordinate increments if a particle moves from $A(4,3)$ to $B(-4,-1)$.

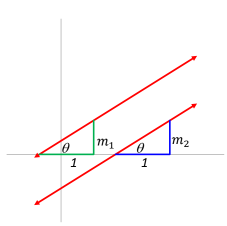
Slope of a Line

Every nonvertical line has a ***slope***.

- $P_1(x_1, y_1) \qquad P_2(x_2, y_2)$
- $rise = \Delta y = y_2 - y_1$
- $run = \Delta x = x_2 - x_1 \neq 0$
- $slope = \frac{rise}{run}$
- $m = \frac{\Delta y}{\Delta x}$

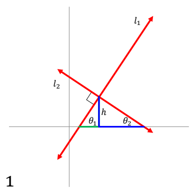
Parallel Lines

- Form equal angles with the *x*-axis.
- $m_1 = m_2$
- Parallel lines have equal slopes.



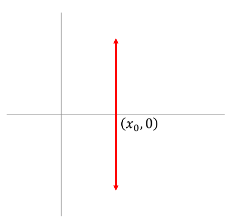
Perpendicular Lines

- Form supplementary angles with the x-axis.
- $\theta_1 + \theta_2 = 90°$
- $m_1 = \tan \theta_1$
- $m_2 = -\tan \theta_2$
- $\tan \theta_2 = \tan(90° - \theta_1) = \frac{1}{\tan \theta_1}$
- $m_1 m_2 = -1$



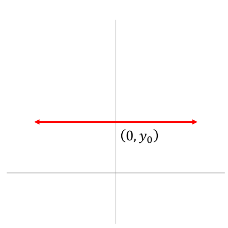
Vertical Lines

- All points have the same x-value.
- Crosses x-axis at x_0.
- $x = x_0$.
- Slope undefined



Horizontal Lines

- All points have the same y-value.
- Crosses y-axis at y_0.
- $y = y_0$.
- $m = 0$



Exercise 2

- Find the equations for the vertical line and the horizontal line passing through the point $P(2,3)$

Point-Slope Form

- The equation of a line having slope m and passing through the point $P(x_0, y_0)$ in point-slope form is,

$$y - y_0 = m(x - x_0)$$

$$y = m(x - x_0) + y_0$$

Exercise 3

- Find the equation of the line with slope $m = 2$ passing through the point $P(1, -1)$.

Slope-Intercept Form

- The y–coordinate of the point where a nonvertical line crosses the y–axis is the y–intercept.
- A line with slope m and passing through the point $P(0, b)$ has the following equation:
  - $y - b = m(x - 0)$

    or
  - $y = mx + b$

x-Intercept

- The x–coordinate of the point where a nonhorizontal line crosses the x–axis is the x–intercept.

Exercise 4

- Write the equation in slope-intercept form for a line passing through the following points:

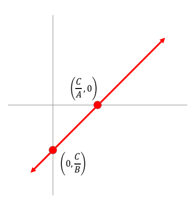
  $(2, -1) \quad (-3, 4)$

General Form

- Also called Standard Form
- $Ax + By = C$
  - Not ***both*** A and B zero

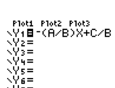
Graphing a General Linear Equation

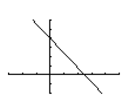
- $Ax + By = C$
- Find the x-intercept
  - $y = 0$
  - $x = \frac{C}{A}$
  - $\left(\frac{C}{A}, 0\right)$
- Find the y-intercept
  - $x = 0$
  - $y = \frac{C}{B}$
  - $\left(0, \frac{C}{B}\right)$



Graphing a General Linear Equation

- To use a graphing Calculator,
  - Transform the linear equation from general form to slope-intercept form
  - Enter it into the equation editor of the graphing calculator





Exercise 5

- Graph the following linear equation:
  $4x + 3y = 12$

Writing Equations

- Write an equation for the line passing through the point $(-1,2)$ that is parallel to the line L: $y = 3x - 4$.
- Slope:
  - $m_L = 3$
  - $m_{\parallel} = m_L = 3$
- $y - 2 = 3\big(x - (-1)\big)$
- $y - 2 = 3(x + 1)$

Writing Equations

- Write an equation for the line passing through the point $(-1,2)$ that is perpendicular to the line L: $y = 3x - 4$.
- Slope:
  - $m_L = 3$
  - $m_{\perp} = -\frac{1}{m_L} = -\frac{1}{3}$
- $y - 2 = -\frac{1}{3}\big(x - (-1)\big)$
- $y - 2 = -\frac{1}{3}(x + 1)$

Exercise 6

- Write equations for the lines passing through the point $(4, -2)$ that are parallel and perpendicular to the line $L: y = 2x - 4$.

Determining Linear Functions

- Given a table of values of a linear function, you can determine the function as follows:
  - Use two of the points to find the slope.
  - Use any point and the slope to find the equation in point-slope form.
  - Transform the equation into slope-intercept form.
  - Replace y with $f(x)$.
- $f(x) = mx + b$.

Exercise 7

- Find the linear function that produced the following table:

x	$f(x)$
0	3
3	9
6	15

Conversions

- Many important quantities are related by a linear function.
- Celsius and Fahrenheit are related by the linear function,

$$C = \frac{5}{9}(F - 32)$$

- Transforming this function,

$$F = \frac{9}{5}C + 32$$

Exercise 8

- The pressure p experienced by a diver under water is related to the diver's depth by d by an equation of the form $p = kd + 1$ (k a constant). When $d = 0$ meters, the pressure is one atmosphere. The pressure at 100 m is 10.94 atmospheres. Find the pressure at 50 m.

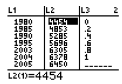
Regression Analysis

- A regression curve is calculated to represent the function associated with data.

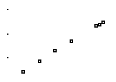
1. Make a scatter plot of the data.
2. Find the regression curve.
   a. For a line, it has the form, $f(x) = mx + b$.
3. Graph the regression curve on the scatter plot to see the fit.
4. Use the regression equation to predict y–values for particular values of x.

Regression Analysis – Example

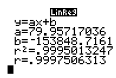
- Enter the data



- Generate a scatter plot

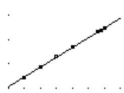


- Perform the regression analysis

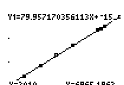


Regression Analysis – Continued

- Graph the regression curve



- Predict the population for 2010



Homework

- Page 9:
- 1-21 every other odd (EOO, 1,5,9,etc.),
- 22, 23, 25-37odds,
- 38-41 all, 43, 44, 47-52 all,
- 54, 55, 57