Algebra II

Name ________________________________

Period __________

Date: **Unit 3: Polynomials and Polynomial Functions** **Lesson 1: Operations with Polynomials**	Essential Question: **You know that multiplication is repeated addition. For example, $3 \cdot 2 = 2 + 2 + 2$. How is exponentiation related to multiplication?**
Standard: A-APR.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
Learning Target:	80% of the students will be able to multiply $(3x^2 + 7x - 2)(2x - 3)$.
Vocabulary:	(see table below)

Definition	***Examples***
Constant: a number	$-2, \frac{3}{5}, 0$
Monomial: a constant, a variable, or a product of a constant and one or more variables	$-7, u, \frac{1}{3}m^2, -s^2t^3$
Coefficient (or ***numerical coefficient***): the constant (or numerical) factor in a monomial	The coefficient of $3w^2$ is **3**.
Degree of a variable in a monomial: the number of times the variable occurs as a factor in the monomial	The degree of *x* is 1. The degree of y^3 is 3.
Degree of a monomial: the sum of the degrees of the variables in the monomial. A nonzero constant has degree 0. The constant 0 has *no degree*.	$6xy^3$ has degree $1 + 3 = 4$.

Summary

Vocabulary continued:

Similar (or ***like***) ***monomials***: monomials that are identical or that differ only in their coefficients	$-s^2t^3$ and $2s^2t^3$ are similar. $6xy^3$ and $6x^3y$ are not similar.
Polynomial: a monomial or a sum of monomials. The monomials in a polynomial are called the ***terms*** of the polynomial.	$x^2 + (-4)x + 5$, or $x^2 - 4x + 5$ The terms are, x^2, $-4x$, and 5.
Simplified polynomial: a polynomial in which no two terms are similar. The terms are usually arranged in order of decreasing degree of one of the variables.	$2x^3 - 5 + 4x + x^3$ is not simplified, but $3x^3 + 4x - 5$ is.
Degree of a polynomial: the greatest of the degrees of its terms after it has been simplified	The degrees of the terms in $x^4 - 2x^2y^3 + 6y - 11$ are, in order, 4, 5, 1, and 0. The polynomial has degree 5.

Simplifying a Polynomial:

When ***simplifying a polynomial***, you use many of the properties of real numbers. By using the commutative and associative properties of addition, you can order and group the terms of a polynomial in any way. Similar terms are usually grouped together and then combined. For example,

$$4s - 3s^2t - s + 5s^2t - 7$$

$$= (-3s^2t + 5s^2t) + (4s - s) - 7$$

$$= 2s^2t + 3s - 7$$

***Example 1*:**	Simplify, arranging terms in order of decreasing degree of *x*. Then write the degree of the polynomial. a. $x - 3x^2 + 8 + x^2 - 2 + 4x$ $= (-3x^2 + x^2) + (x + 4x) + (8 - 2)$ $= -2x^2 + 5x + 6$ The degrees of the terms are, in order, 2, l, and 0. ∴ the degree of the polynomial is 2. b. $x^3y^3 - 6xy^4 + 2x^3y - x^3y^3 + 3xy^4 - 4x^2y$ $= (x^3y^3 - x^3y^3) + 2x^3y - 4x^2y + (-6xy^4 + 3xy^4)$ $= 0 + 2x^3y - 4x^2y - 3xy^4$ $= 2x^3y - 4x^2y - 3xy^4$ The degrees of the terms are, in order, 4, 3, and 5. ∴ the degree of the polynomial is 5.
***Exercise 1*:**	Simplify, arranging terms in order of decreasing degree of *x*. Then write the degree of the polynomial. a. $2x^3 - 7 + 5x^2 - x^3 + 3x - x^3$

b. $7xy^2z^3 - 4xy^2z^3 + 2x^2yz^2 - 3xy^2z^3$

***Polynomial Addition and Subtraction*:**

Adding and Subtracting Polynomials

To add two or more polynomials, write their sum and then simplify by combining similar terms.

To subtract one polynomial from another, add the opposite of each term of the polynomial you're subtracting.

***Example 2*:**

Add $2x^2 - 3x + 5$ and $x^3 - 5x^2 + 2x - 5$.

***Solution 1*:**

$$2x^2 - 3x + 5 + x^3 - 5x^2 + 2x - 5$$

$$= x^3 + [2x^2 + (-5x^2)] + (-3x + 2x) + [5 + (-5)]$$

$$= x^3 - 3x^2 - x$$

***Solution 2*:**

You can also add vertically.

$$\begin{array}{r} 2x^2 - 3x + 5 \\ x^3 - 5x^2 + 2x - 5 \\ \hline x^3 - 3x^2 - x \end{array}$$

***Exercise 2*:**	Add $4a^2 + 3ab - b^2$ and $b^2 - 2ab$. Add them both vertically and horizontally.
***Example 3*:**	Subtract $2x^2 - 3x + 5$ from $x^3 - 5x^2 + 2x - 5$.
***Solution 1*:**	$(x^3 - 5x^2 + 2x - 5) - (2x^2 - 3x + 5)$ $(x^3 - 5x^2 + 2x - 5) + (-2x^2 + 3x - 5)$ $= x^3 + [-5x^2 + (-2x^2)] + (2x + 3x) + [-5 + (-5)]$ $= x^3 - 7x^2 + 5x - 10$
***Solution 2*:**	You can also subtract vertically. $$\begin{array}{r} x^3 - 5x^2 + 2x - 5 \\ -(2x^2 - 3x + 5) \\ \hline x^3 - 7x^2 + 5x - 10 \end{array}$$

***Exercise 3*:**	Subtract $b^2 - 2ab$ from $4a^2 + 3ab - b^2$. Subtract them both vertically and horizontally.
***Example 4*:**	Simplify $x(2y-3) + 4(x+2y) - 3y(x-1)$ Use the distributive property first. Then combine similar terms. $= 2xy - 3x + 4x + 8y - 3xy + 3y$ $= (-3x + 4x) + (2xy - 3xy) + (8y + 3y)$ $= x - xy + 11y$
***Exercise 4*:**	Simplify $4(3y^2 - 2y) + 3(y^2 + 5y - 1)$

Laws of Exponents:	The properties of exponents are summarized in the following table:

Laws of Exponents

$$\forall\, m > 0, n > 0 \in \mathbb{Z} \text{ and } \forall\, a \neq 0, b \neq 0 \in \mathbb{R}$$

1. $a^m \cdot a^n = a^{m+n}$	Product of Powers
2. $(ab)^m = a^m \cdot b^m$	Power of a Product
3. $(a^m)^n = a^{mn}$	Power of a Power
4. $m > n \Rightarrow \frac{a^m}{a^n} = a^{m-n}$	Quotient of Powers
5. $n > m \Rightarrow \frac{a^m}{a^n} = \frac{1}{a^{n-m}}$	Quotient of Powers
6. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	Power of a Quotient
7. $a^{-n} = \frac{1}{a^n}$	Negative Exponent
8. $a^0 = 1$	Zero Power

First Law:

Without the first law you would have to simplify a product of powers, such as $c^5 \cdot c^3$ by counting factors:

$$c^5 \cdot c^3 = \underbrace{(\overbrace{c \cdot c \cdot c \cdot c \cdot c}^{\text{5 factors}})(\overbrace{c \cdot c \cdot c}^{\text{3 factors}})}_{\text{8 factors}} = c^8$$

By using the first law you can write

$$c^5 \cdot c^3 = c^{5+3} = c^8$$

Second Law:

Using the second law you can simplify the power of a product:

$$(2x)^3 = 2^3x^3 = 8x^3$$

Third Law:

Using the third law you can simplify a power of a power:

$$(10^3)^2 = 10^{3 \cdot 2} = 10^6$$

Proof of Second Law:

We'll prove the second law by counting factors:

1.	$(ab)^m = (ab)(ab)(ab) \cdots (ab)(ab)$	Definition of a power
2.	$= (a \cdot a \cdot \cdots \cdot a)(b \cdot b \cdot \cdots \cdot b)$	Commutative and associative properties of multiplication
3.	$= a^m b^m$	

You can use the laws of exponents along with the commutative and associative properties of multiplication to simplify products and powers like the following examples.

***Example 5*:**	Simplify a. $(-3x^2y^3)(4xy^2)$ Use the first law of exponents and the fact that $x = x^1$. $(-3x^2y^3)(4xy^2) = (-3 \cdot 4)(x^2 \cdot x)(y^3 \cdot y^2)$ $= -12x^3y^5$ b. $(st^4)^3$ Use the second law of exponents then apply the third. $(st^4)^3 = s^3(t^4)^3$ $= s^3t^{12}$ c. $(-x^3)^2$ Use the fact that $-x^3 = (-1)x^3$ and the second law of exponents. $(-x^3)^2 = [(-1)x^3]^2$ $= (-1)^2(x^3)^2$ $= x^6$

***Exercise 5*:**	Simplify a. $(-2u^2)(uv^3)(-u^2v^2)$ b. $(-3pq^4r^2)^3$ c. $(-t^4)^3$

***Example 6*:**	Simplify a. $u \cdot (u^2)^3 \cdot u^5$ $u(u^2)^3u^5 = u \cdot u^6 \cdot u^5$ $= u^{1+6+5} = u^{12}$ b. $(3xy^2z^3)^3$ $(3xy^2z^3)^3 = 3^3x^3(y^2)^3(z^3)^3$ $= 27x^3y^6z^9$
***Exercise 6*:**	Simplify a. $(-z^3)(-z)^3$ b. $(4a^3b^2)^2$

	To multiply a polynomial by a binomial, use the distributive property.
***Example 7*:**	Simplify $(3t^2 + t)(t^3 - 2t^2 + t - 4)$ $(3t^2 + t)(t^3 - 2t^2 + t - 4)$ $= (3t^2)t^3 + (3t^2)(-2t^2) + (3t^2)t + (3t^2)(-4)$ $+ (t)t^3 + (t)(-2t^2) + (t)t + t(-4)$ $= 3t^5 - 6t^4 + 3t^3 - 12t^2$ $+ t^4 - 2t^3 + t^2 - 4t$ $= 3t^5 - 5t^4 + t^3 - 11t^2 - 4t$
***Exercise 7*:**	Simplify a. $(3y - 1)(y^3 - 2y^2 + 3)$ b. $(rs^2 + 2s)(r^2 - 2rs - s^2)$

***Example 8*:**	Simplify. Assume that variable exponents represent positive integers. a. $(a^2)^k(a^k)^3$ $(a^2)^k(a^k)^3 = a^{2k} \cdot a^{3k}$ $= a^{2k+3k}$ $= a^{5k}$ b. $x^{m-n}(x^{m+n} + x^n)$ $x^{m-n}(x^{m+n} + x^n) = x^{m-n} \cdot x^{m+n} + x^{m-n} \cdot x^n$ $= x^{m-n+m+n} + x^{m-n+n}$ $= x^{2m} + x^m$

***Exercise 8*:**	Simplify. Assume that variable exponents represent positive integers. a. $x^{m-1} \cdot x \cdot x^m$ b. $p^n(p^{m-n+1} + p^{m-n})$
Class work:	p 307: 1-15
Homework:	p 307: 17-27 odd, 29, 33, 37, 40, 41, 45, 49, 53, 57, 61