Algebra II

Name ______________________________

Period __________

Date: Chapter: **2: Quadratic Functions and Relations** Section: **8: The Quadratic Formula and the Discriminant**	Essential Question: **How do the values of *a*, *b*, and *c* in the quadratic equation influence its solution?**
Standard: A-REI.4a	Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
Learning Target:	Learn how to solve quadratic equations using the quadratic formula. 80% of the students will be able to use the quadratic formula to solve $3x^2 - 5x = 7$.
Derivation of quadratic formula	Quadratic equations are used in many applications, so it is useful to have a formula that gives their solutions directly from the coefficients. You can derive this formula by applying the method of completing the square to the general quadratic equation: $ax^2 + bx + c = 0$ $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$ $x^2 + \frac{b}{a}x = -\frac{c}{a}$
Summary	

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac + b^2}{4a^2}$$

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The Quadratic Formula

The solutions of the quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ are given by the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 1	Solve $3x^2 + x - 1 = 0$
Solution	$a = 3$ $b = 1$ $c = -1$ $x = \frac{-1 \pm \sqrt{(1)^2 - 4(3)(-1)}}{2(3)}$ $x = \frac{-1 \pm \sqrt{1 - (-12)}}{6} = \frac{-1 \pm \sqrt{13}}{6}$ $\therefore$ the solution set is $\left\{\frac{-1+\sqrt{13}}{6}, \frac{-1-\sqrt{13}}{6}\right\}$. In practical applications, you often need an approximate answer. Using a calculator and rounding to the nearest hundredth, the solution set is, $\{0.43, -0.77\}$.
Exercise 1:	Solve $2x^2 - 3x + 1 = 0$ $a =$ $b =$ $c =$ $x = \frac{-(\quad) \pm \sqrt{(\quad)^2 - 4(\quad)(\quad)}}{2(\quad)}$ $x =$

***Exercise 1 continued*:**	Solve $3x^2 + 7x - 5 = 0$
***Example 2*:**	Solve $5y^2 = 6y + 3$ $5y^2 - 6y - 3 = 0$ $a = 5$ $b = -6$ $c = -3$ $y = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(5)(-3)}}{2(5)}$ $y = \frac{6 \pm \sqrt{96}}{10} = \frac{3 \pm 2\sqrt{6}}{5}$ $\therefore$ the solution set is, $\left\{\frac{3 + 2\sqrt{6}}{5}, \frac{3 - 2\sqrt{6}}{5}\right\}$

	In Examples 1 and 2, the coefficients a, b, and c were integers. Keep in mind, however, that the quadratic formula can be used to solve any quadratic equation, whether the coefficients are integers, fractions, decimals, or irrational numbers.
Exercise 2:	Solve $x^2 = 4 - 2x$
Discriminant:	Using the quadratic formula, you can write the two roots of the quadratic equation, $ax^2 + bx + c = 0$, as follows: $$r_1 = \frac{-b + \sqrt{D}}{2a}$$ and $$r_2 = \frac{-b - \sqrt{D}}{2a}$$ Where $$D = b^2 - 4ac$$ D is known as the ***discriminant***.

***Example 3*:**	Find the discriminant of the following quadratic equations: 1. $x^2 + 6x - 2 = 0$ $a = 1$ $b = 6$ $c = -2$ $D = 6^2 - 4 \cdot 1 \cdot (-2)$ $= 36 + 8 = 44$ 2. $3x^2 - 4\sqrt{3}x + 4 = 0$ $a = 3$ $b = -4\sqrt{3}$ $c = 4$ $D = \left(-4\sqrt{3}\right)^2 - 4 \cdot 3 \cdot 4$ $= 48 - 48 = 0$ 3. $x^2 - 6x + 10 = 0$ $a = 1$ $b = -6$ $c = 10$ $D = 6^2 - 4 \cdot 1 \cdot 10$ $= 36 + 40 = -4$

***Exercise 3*:**	Find the discriminant of the following quadratic equations: 1. $x^2 + 6x + 3 = 0$ 2. $x^2 + 8x + 16 = 0$ 3. $3x^2 - 4x + 2 = 0$

***Example 4*:**	Use the discriminant to find the two roots of the quadratic equations from Example 3: 1. $x^2 + 6x - 2 = 0$ $a = 1 \quad b = 6 \quad c = -2$ $D = 44$ $r_1 = \frac{-6 + \sqrt{44}}{2 \cdot 1} = -3 + \sqrt{11}$ $r_2 = \frac{-6 - \sqrt{44}}{2 \cdot 1} = -3 - \sqrt{11}$ 2. $3x^2 - 4\sqrt{3}x + 4 = 0$ $a = 3 \quad b = -4\sqrt{3} \quad c = 4$ $D = 0$ $r_1 = \frac{4\sqrt{3} + \sqrt{0}}{2 \cdot 3} = \frac{2\sqrt{3}}{3}$ $r_2 = \frac{4\sqrt{3} - \sqrt{0}}{2 \cdot 3} = \frac{2\sqrt{3}}{3}$ 3. $x^2 - 6x + 10 = 0$ $a = 1 \quad b = -6 \quad c = 10$ $D = -4$ $r_1 = \frac{6 + \sqrt{-4}}{2 \cdot 1} = -3 + i$ $r_2 = \frac{6 - \sqrt{-4}}{2 \cdot 1} = -3 - i$

Exercise 4:	Use the discriminant to find the two roots of the quadratic equations from Exercise 3: 1. $x^2 + 6x + 3 = 0$ 2. $x^2 + 8x + 16 = 0$ 3. $3x^2 - 4x + 2 = 0$

***The Nature of the Roots of a Quadratic Equation*:**	Let $ax^2 + bx + c = 0$ be a quadratic equation with ***real*** coefficients. 1. $b^2 - 4ac > 0 \Rightarrow$ there are two unequal real roots. 2. $b^2 - 4ac = 0 \Rightarrow$ there is a real double root. 3. $b^2 - 4ac < 0 \Rightarrow$ there are two conjugate imaginary roots. $D = b^2 - 4ac$ is called the discriminant, because it discriminates among the three cases of the roots of the quadratic equation.
***Example 5*:**	Determine whether the roots of the following quadratic equations are rational or irrational: 1. $x^2 + 5x + 4 = 0$ $a = 1 \quad b = 5 \quad c = 4$ $D = 5^2 - 4 \cdot 1 \cdot 4 = 9 = 3^2$ $r_1 = \frac{-5 + \sqrt{9}}{2 \cdot 1} = \frac{-5 + 3}{2} = -1$ $r_2 = \frac{-5 - \sqrt{9}}{2 \cdot 1} = \frac{-5 - 3}{2} = -4$ rational 2. $3x^2 - 4x - 3 = 0$ $a = 3 \quad b = -4\sqrt{3} \quad c = -3$ $D = 52 = 2\sqrt{13}$ $r_1 = \frac{4 + \sqrt{52}}{2 \cdot 3} = \frac{2 + \sqrt{13}}{3}$ $r_2 = \frac{4 - \sqrt{52}}{2 \cdot 3} = \frac{2 - \sqrt{13}}{3}$ Irrational

***Exercise 5*:**	First calculate the discriminant, then determine whether the roots of the following quadratic equations are rational or irrational: 1. $2x^2 - 13x + 15 = 0$ 2. $x^2 - 5x - 5 = 0$ 3. $\sqrt{5}x^2 - 6x + \sqrt{5} = 0$
Class work: p 269: 1-13	
Homework: p 270: 14-18, 20, 33, 34, 41, 42	