Algebra II

Name _____

Period _____

Date: Unit 2: **Quadratic Equations** Lesson 4: **Solving Quadratic Equations by Factoring, Part 2**	Essential Question: **You can factor quadratic trinomials by trial and error. How does factoring by grouping compare to trial and error?**
Standard: A-REI.4b	Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation.
***Learning Target*:** ***Real-Life Example*:**	The student will learn to factor quadratic polynomials when the quadratic coefficient does not equal 1. 80% of the students will be able to factor, $6x^2 - 5x - 6.$ Bungee jumping has become an activity for thrill seeking individuals. The first known bungee jumpers were inhabitants of south Pacific islands.[1]  "In the times when legends were made, a man named Tamalie frequently and viciously beat his wife. One day she ran away in fear and hid in the top of a tree. Tamalie climbed after her despite threats she would jump. When he reached the top, she threw herself off and he plunged after her. However, she had cleverly tied vines to her feet that broke her fall while her husband crashed to his death on the ground."[2]
Summary	

[1] Land diving on Pentecost Island , Vanuatu. Photo courtesy of www.travellerspoint.com

[2] https://suite.io/bruce-iliff/2fn22x3

	If the tree were 74 ft tall and Tamalie jumped with an initial downward velocity of 5 ft per second, How long was he in the air before he hit the ground? This problem can be modeled by the following equation: $h = -16t^2 - 5t + 74$ where h is his distance above the ground, and t is the time after he jumped. When Tamalie hit the ground, the height was $h = 0$. $0 = -16t^2 - 5t + 74$ The problem reduces to solving this quadratic equation: $-16t^2 - 5t + 74 = 0$ $16t^2 + 5t - 74 = 0$ Multiply by –1 We have factored quadratic trinomials that had leading coefficients of 1, but this quadratic expression has a leading coefficient of 16. Can we find a way to use our previous techniques to factor this quadratic trinomial?

***General Quadratic Polynomials*:**	Polynomials of the form, $ax^2 + bx + c \qquad a \neq 1,$ are ***general quadratic polynomials***. We have learned how to use grouping to factor quadratic trinomials when the leading coefficient is 1. Now, we will extend that technique to factoring general quadratic trinomials.
***Positive a*:**	We will always assume that the quadratic coefficient, *a*, is positive. From a practical standpoint, we can always change the signs of all the coefficients in order to make *a* positive.
***FOIL*:**	Let's use the FOIL technique to multiply the following binomials: $(a_1x + c_1)(a_2x + c_2)$
***First*:**	$a_1a_2x^2$
***Outer*:**	a_1c_2x
***Inner*:**	a_2c_1x
***Last*:**	c_1c_2
	Notice that $a = a_1a_2$ $b = a_1c_2 + a_2c_1$ $c = c_1c_2$ Furthermore, $ac = a_1a_2c_1c_2$ $= (a_1c_2)(a_2c_1)$ The linear coefficient, *b*, is the sum of the factors of *ac*. This is the requirement to factor the quadratic trinomial by grouping. We shall illustrate this approach in the following examples and exercises:

<table>
<tr><td colspan="2">Define the following terms.</td></tr>
<tr><td>Quadratic Polynomials</td><td></td></tr>
<tr><td>Standard Form of Quadratic Polynomials</td><td></td></tr>
<tr><td>FOIL</td><td></td></tr>
<tr><td>Factor Quadratic Polynomial</td><td></td></tr>
<tr><td>Factor by Grouping</td><td></td></tr>
<tr><td colspan="2">Now go back and paraphrase page 4.</td></tr>
<tr><td>Example 1:</td><td>Factor the following quadratic trinomial by grouping.

$3x^2 + 7x + 2$

The product of the quadratic and constant coefficients is

$ac = 3 \cdot 2 = 6$

Let’s make a table of all the integral factors of 6 and their sums.
<table><tr><th>b_1</th><th>b_2</th><th>$b_1 + b_2$</th></tr><tr><td>1</td><td>6</td><td>7</td></tr><tr><td>2</td><td>3</td><td>5</td></tr></table>
$1 \cdot 6 = 6$ $1 + 6 = 7$</td></tr>
</table>

	Therefore, we can factor this quadratic trinomial by grouping. First, we write, $3x^2 + 7x + 2 = 3x^2 + 1 \cdot x + 6x + 2$ Then we group the terms and factor them. $3x^2 + 7x + 2 = (3x^2 + 1 \cdot x) + (6x + 2)$ $= x(3x + 1) + 2(3x + 1)$ $= (x + 2)(3x + 1)$ This can easily be checked by using FOIL.
***Exercise 1*:**	Factor the following quadratic trinomial by grouping. $4x^2 + 8x + 3$

***Example 2*:**

If the linear coefficient is negative, and the constant is positive, then we must look for all the negative factors of *ac*.

Factor the following quadratic trinomial by grouping.

$$8x^2 - 22x + 5$$

The product of the quadratic and constant coefficients is

$$ac = 8 \cdot 5 = 40$$

Let's make a table of all the negative, integral factors of 40 and their sums.

b_1	b_2	$b_1 + b_2$
–1	–40	–41
–2	–20	–22
–4	–10	–14
–5	–8	–13

$$-2 \cdot (-20) = 40 \qquad -2 + (-20) = -22$$

Therefore, we can factor this quadratic trinomial by grouping. First, we write,

$$8x^2 - 22x + 5 = 8x^2 - 2x - 20x + 5$$

Then we group the terms and factor them.

$$8x^2 - 22x + 5 = (8x^2 - 2x) + (-20x + 5)$$

$$= 2x(4x - 1) - 5(4x - 1)$$

$$= (2x - 5)(4x - 1)$$

As before, we can easily check this by using FOIL.

***Exercise 2*:**	Factor the following quadratic trinomial by grouping. $4x^2 - 17x + 15$
	If the constant coefficient is negative, then we must look for all the factors of *ac* that result in a negative product. That is, one factor must be negative, and the other must be positive.
***Example 3*:**	Factor the following quadratic trinomial by grouping. $2x^2 - 3x - 5$ The product of the quadratic and constant coefficients is $ac = 2 \cdot (-5) = -10$ On the next page, let's make a table of all the integral factors of –10 and their sums.

b_1	b_2	$b_1 + b_2$
–1	10	9
–2	5	3
–5	2	–3
–10	1	–9

$-5 \cdot 2 = -10$ $\qquad$ $-5 + 2 = -3$

Therefore, we can factor this quadratic trinomial by grouping. First, we write,

$$2x^2 - 3x - 5 = 2x^2 - 5x + 2x - 10$$

Then we group the terms and factor them.

$$2x^2 - 3x - 5 = (2x^2 - 5x) + (2x - 5)$$

$$= x(2x - 5) + 1 \cdot (2x - 5)$$

$$= (x + 1)(2x - 5)$$

We can easily check this by using FOIL.

***Exercise 3*:** Factor the following quadratic trinomial by grouping.

$$6x^2 + 5x - 6$$

***Real-Life Example*:**

Let's revisit the story of Tamalie.

As we saw, this problem reduces to solving this quadratic equation:

$$16t^2 + 5t - 74 = 0$$

In order to factor this quadratic expression, we must find the factors of $16 \cdot (-74) = -1184$.

Moreover, the product is negative. Therefore, one factor must be negative, and the other must be positive.
The sum of the two factors must be 5; the absolute values of the two factors must be close to $\sqrt{1184} \cong 34.4$. Let's check the negative integers between -30 and -34 and the positive integers that would give a sum of 5.

n	$5 - n$	$n(5 - n)$
-30	35	-1050
-31	36	-1116
-32	37	-1184
-33	38	-1254
-34	39	-1326

We see that,

$$-32 + 37 = 5 \qquad \text{and} \qquad (-32) \cdot 37 = -1184$$

We can rewrite the quadratic equation as,

$$16t^2 - 32t + 37t - 74 = 0$$

$$(16t^2 - 32t) + (37t - 74) = 0$$

$$16t(t - 2) + 37(t - 2) = 0$$

$$(16t + 37)(t - 2) = 0$$

	Therefore, the solutions are, $(16t + 37) = 0 \quad \therefore \quad t = -{}^{37}/_{16}$ s. and $(t - 2) = 0 \quad \therefore \quad t = 2$ s. The first solution is negative, which means it is before Tamalie jumped. It is a solution of the equation, but it is not consistent with the physical problem. It is an ***extraneous*** solution. When you model a physical problem with an equation, you must always check for extraneous solutions. You must reject any extraneous solution. The other solution is, $t = 2$ s. Tamalie was in the air for two seconds before he fatally crashed into the ground.
Class work:	p 242: 1-5, 8, 9, 11, 12, 14, 16
Homework:	p 242: 17-25 odd, 29-37 odd, 44, 49-65 odd, 66-68, 71-79 odd