Algebra II

Name ________________________________

Period __________

Date:

Unit 2: **Quadratic Relations and Functions**

Lesson 3: **Solving Quadratic Equations by Factoring Part 1**

Essential Question: **How is factoring like dividing?**

Standard: A-REI.4b

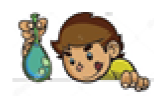
Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation.

***Learning Target*:**

The student will learn to factor quadratic polynomials when the quadratic coefficient equals 1. 80% of the students will be able to factor the following quadratic expression:

$$x^2 - 2x - 3$$

Willy is standing on the edge of a cliff. He is holding a water balloon 29 feet above a path that runs by the base of the cliff. If he throws the balloon straight down with a downward velocity of 8 ft/s, its height will be given by the following equation: $h = -16t^2 - 8t + 29$, where t is the time after he throws the balloon.



Rhoda is jogging along the path towards Willy. Willy thinks he wants to throw the balloon so that it hits Rhoda on the top of her head. Rhoda is 5 feet tall, and she is jogging at 8 feet per second. The distance Rhoda travels is given by the rate equation, $d = 8t$. How far from the spot on the road directly below Willy should Rhoda be when Willy throws the balloon if he wants it to hit Rhoda on the top of her head?

Summary

	First, we must find how long the balloon will take to reach 5 feet. $$5 = -16t^2 - 8t + 29$$ or $$16t^2 + 8t - 24 = 0$$ In that time, Rhoda will jog a distance of, $d = 8t$ feet. or $$t = \frac{d}{8}$$ Substituting this into the quadratic equation above gives, $$16\left(\frac{d}{8}\right)^2 + 8\frac{d}{8} - 24 = 0$$ $$\frac{1}{4}d^2 + d - 24 = 0$$ Multiplying by 4 removes the fraction, $$d^2 + 4d - 96 = 0$$ Now, all Willy has to do is to solve this equation, and he will know the distance Rhoda should be from the spot directly below him when he throws the balloon. In this lesson, we will learn how to solve this type of equation. First, let's define some terms. Use your smart phones, tablets, Chrome books, the computers in the classroom, or whatever technology you have to find the definitions of the terms on the next page.

Define the following terms	
Polynomial	
Quadratic Polynomial	
Second Degree Polynomials	
Trinomial	
Perfect Square Trinomials	
Quadrinomial	
Factor	
Factoring by Grouping	

Quadratic Trinomials:	Polynomials of the form, $ax^2 + bx + c \qquad a \neq 0,$ are ***quadratic polynomials***. Since the expression has three terms, we call them ***quadratic trinomials***. We also call them ***second degree polynomials***. (The degree of the monomial with the highest degree is 2.)
Quadratic Trinomials in Factored Form:	If we can write the quadratic trinomial in factored form, then solving the quadratic equation becomes easy. For example, how would we solve $x^2 - 5x + 6 = 0$? When we notice that $x^2 - 5x + 6 = (x - 2)(x - 3)$, the solution becomes easy. If $(x - 2)(x - 3) = 0$ then either $x - 2 = 0$ or $x - 3 = 0$. The solution set is $\{2, 3\}$. In this lesson we will learn how to determine if $x^2 + bx + c$ can be factored and how to find those factors.
Quadratic Quadrinomials:	2nd degree polynomial with four terms $a \neq 0 \qquad b_1 \neq 0 \qquad b_2 \neq 0 \qquad c \neq 0$ ***a*:** *leading coefficient* ***ax^2*:** *leading term* ***b_1*:** *first linear coefficient* ***b_1x*:** *first linear term* ***b_2*:** *second linear coefficient* ***b_2x*:** *second linear term* ***c*:** *constant coefficient* ***c*:** *constant term, constant*

	In this lesion, we will concentrate on quadratic polynomials that have the leading coefficient equal to 1. That is, $$a = 1$$ Under the right circumstances, such a quadratic quadrinomial can be factored by grouping.
***Factoring by Grouping*:**	Suppose $c = b_1 \cdot b_2$ then the quadrinomial, $x^2 + b_1x + b_2x + c$ can be written, $x^2 + b_1x + b_2x + b_1b_2$ This can be rewritten, $(x^2 + b_1x) + (b_2x + b_1b_2)$ We can factor the first group and the second group: $x(x + b_1) + b_2(x + b_1)$ Using the distributive property, $(x + b_2)(x + b_1)$ We have successfully factored the original quadratic quadrinomial.
***Requirement*:**	***If the product of the two linear terms equals the constant term, the quadratic quadrinomial can be factored by grouping.*** $$\boldsymbol{b_1b_2 = c}$$

***Example 1*:**	Factor the following quadratic quadrinomial by grouping: $x^2 + 3x - 5x - 15$ 1. Check the conditions for factoring by grouping. $3 \cdot (-5) = -15$ 2. Group the terms into pairs that have a common factor. $(x^2 + 3x) + (-5x - 15)$ 3. Factor out the common factors. $x(x + 3) - 5(x + 3)$ 4. Apply the distributive property. $(x - 5)(x + 3)$
***Exercise 1*:**	Factor the following quadratic quadrinomials by grouping: a. $x^2 - 4x - 5x + 20$ b. $x^2 + 3x + x + 3$ c. $x^2 - 7x + 3x - 21$

	We do not often have quadratic quadrinomials to factor. Nevertheless, we can often transform a quadratic trinomial into a quadratic quadrinomial in order to factor it.
Quadratic Trinomial:	2^{nd} degree polynomial with three terms $a \neq 0$ $b \neq 0$ $c \neq 0$ ***a*****:** *leading coefficient* ***ax*****²:** *leading term* ***b*****:** *linear coefficient* ***bx*****:** *linear term* ***c*****:** *constant coefficient* ***c*****:** *constant term, constant*
Transforming Quadratic Trinomials:	If we can find two numbers, b_1, and b_2, such that, $b_1 \cdot b_2 = c$ and $b_1 + b_2 = b$, we can transform the quadratic trinomial into a quadratic quadrinomial and factor it.

***Example 2*:**	Transform $x^2 + 2x - 15$ into a quadratic quadrinomial and factor it.

Let's make a table of all the integral factors of -15 and their sums.

$\boldsymbol{b_1}$	$\boldsymbol{b_2}$	$\boldsymbol{b_1 + b_2}$
-1	15	14
-3	5	2
-5	3	-2
-15	1	-14

From the table, we can see that if

$b_1 = -3$ and $b_2 = 5,$

then

$b_1 \cdot b_2 = -15$ and $b_1 + b_2 = 2.$

We can transform the quadratic trinomial into the following quadratic quadrinomial and factor it:

$$\begin{aligned} x^2 + 2x - 15 &= x^2 - 3x + 5x - 15 \\ &= (x^2 - 3x) + (5x - 15) \\ &= x(x - 3) + 5(x - 3) \\ &= (x + 5)(x - 3) \end{aligned}$$

***Exercise 2*:**	Transform the following quadratic trinomials into quadratic quadrinomials, and then factor them. a. $x^2 + 6x + 5$ b. $x^2 + 5x - 36$ c. $x^2 - 12x + 20$

Define the following term:	
***Zero Product Property*:**	
***Solving Quadratic Equations by Factoring*:**	Consider the following quadratic equation: $x^2 + b_1x + b_2x + b_1b_2 = 0$ We know that the quadratic expression on the left of the equal sign can be factored: $(x + b_1)(x + b_2) = 0$ The zero product property tells us that if either of the factors equals zero, then the product equals zero. Therefore, the solution for either of these equations is a solution for the original equation: $(x + b_1) = 0$ $(x + b_2) = 0$ Specifically, $x = -b_1$ and $x = -b_2$ are solutions for the original equation.

***Example 3*:**	Solve the following equation by factoring: $x^2 - 3x - 18 = 0$ The constant term, -18, has six possible combinations of integral factors: $(-1) \times (18)$, $(-2) \times (9)$, $(-3) \times (6)$, $(1) \times (-18)$, $(2) \times (-9)$, and $(3) \times (-6)$, The sum of the last pair is $(3) + (-6) = -3$. Therefore, we can group the quadratic expression as follows: $x^2 + 3x - 6x - 18 = 0$, and factor it: $(x^2 + 3x) + (-6x - 18) = 0$ $x(x + 3) - 6(x + 3) = 0$ $(x - 6)(x + 3) = 0$ The zero product property tells us that if either factor is zero, the equation is solved: $(x - 6) = 0$ or $(x + 3) = 0$ Therefore, the solutions are, $x = 6$ and $x = -3$

Exercise 3:	Solve the following equations by factoring: a. $x^2 + 3x + 2 = 0$ b. $x^2 - 8x + 15 = 0$ c. $x^2 - x = 12$ Now solve Willy's problem, $d^2 + 4d - 96 = 0$
Class work:	p 242: 6, 7, 10, 13, 14
Homework:	p 242: 26, 27, 38-41, 45, 47, 69