Algebra II

Name ________________________________

Period __________

Date: Unit **2: Quadratic Functions** Lesson **2: Solving Quadratic Equations by Graphing**	Essential Question: **What is the process for writing a quadratic equation that has solutions *x* = 3 and *x* = 7?**
Standard: F-IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*
***Learning Target*:**	Graph quadratic functions. Find the roots, and estimate the solutions of the quadratic equation. 80% of the students will be able to graph the following function, and determine the solutions of the quadratic equation, $f(x) = 0$: $f(x) = 3x^2 - 4x - 4.$
***Quadratic Equation*:**	A ***quadratic equation*** is a quadratic function that has been set to some constant value. For example, the ***function*** $f(x) = 2x^2 - 3x + 4$ becomes a quadratic ***equation*** when we set it equal to some constant, say, 5. $f(x) = 5$, or $5 = 2x^2 - 3x + 4.$ By subtracting 5 from both sides, we can rewrite this equation as, $2x^2 - 3x - 1 = 0.$ This is a quadratic equation in standard form.
Summary	

***Standard Form*:**	Customarily, we choose $f(x) = 0$ to create the quadratic equation. Then solving the quadratic equation reduces to finding the roots of the quadratic function. The standard form of the quadratic equation is, $ax^2 + bx + c = 0,$ **where, $a \neq 0$, and a, b, and c are integers.**
***Converting to Standard Form*:**	Suppose you had a quadratic equation that contained a fractional coefficient, such as, $x^2 - \frac{1}{2}x + 2 = 0$ How would you convert this equation to standard form?
Define the following terms:	
Quadratic Function	
Quadratic Equation	

Root of a Quadratic Function	
Solution of a Quadratic Equation	
***Example 1*:**	Find the solutions of the following quadratic equation by graphing: $x^2 - 4x - 5 = 0$ First, graph the quadratic function, $f(x) = x^2 - 4x - 5$ 1. Find the *y*-intercept. The *y*-intercept is the point where the curve crosses the *y*-axis: (0, *c*). *y*-intercept: (0, −5). 2. Find the vertex. The axis of symmetry: $x = -\frac{b}{2a} = 2.$ *y*-value of the vertex: $y = f(2) = -9$ vertex: (2, −9).

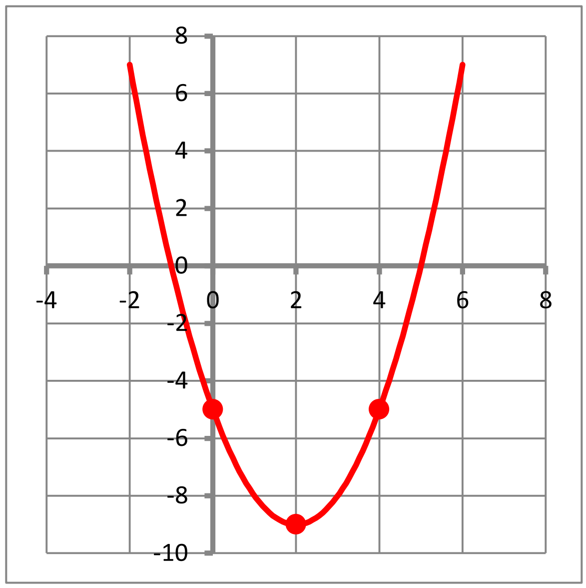
3. Find the reflection of the *y*-intercept about the axis of symmetry.

   reflection: (4, –5).

4. Graph these three points.

5. Draw a smooth curve that crosses the *x*-axis.

Sometimes, you may find it useful to graph another point, and its mirror image on the other side of the *x*-axis.



The roots appear to be, $(-1{,}0)$ and $(5{,}0)$. Let's substitute them into the quadratic equation.

$$x^2 - 4x - 5 = 0$$

$x = -1 \Rightarrow$

$$(-1)^2 - 4(-1) - 5 = 1 + 4 - 5 = 0 \quad ✓$$

$x = 5 \Rightarrow$

$$(5)^2 - 4(5) - 5 = 25 - 20 - 5 = 0 \quad ✓$$

The solutions of the quadratic equation

$x^2 - 4x - 5 = 0$ are,

$x = -1$ and $x = 5$.

Exercise 1**:**

Find the solutions of the following quadratic equation by graphing:

$$x^2 + 4x + 3 = 0$$

1. Find the *y*-intercept.

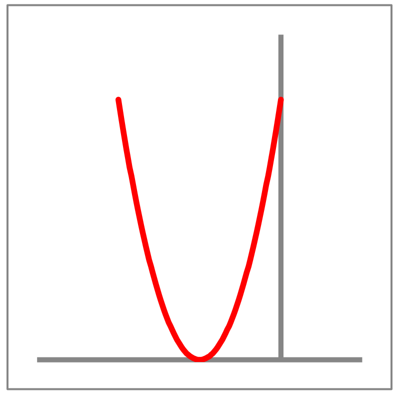
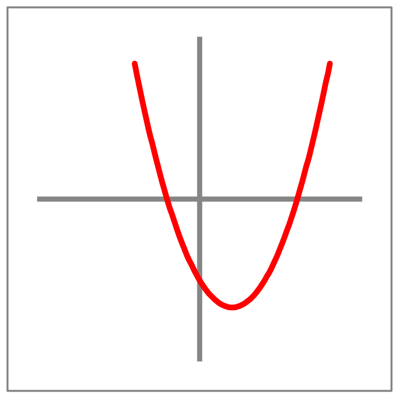
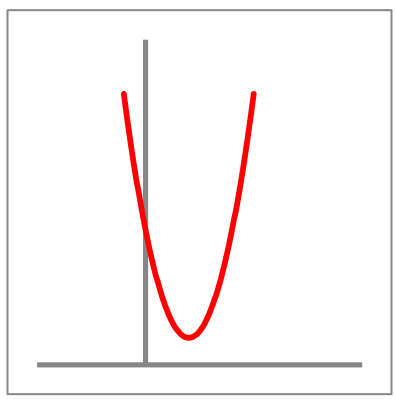
2. Find the vertex.

3. Find the reflection of the *y*-intercept about the axis of symmetry.

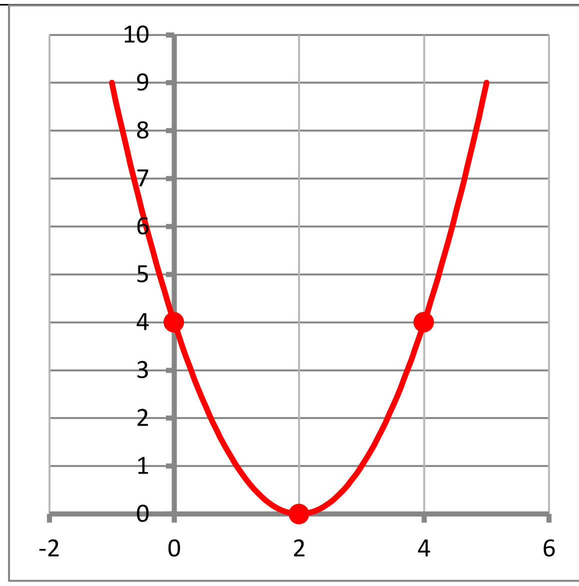
4. Graph these three points.

5. Draw a smooth curve that crosses the *x*-axis.

	Estimate the solutions and check them with the original quadratic equation.
Exercise 1 continued:	Find the solutions of the following quadratic equation by graphing: $-x^2 + 4x + 12 = 0$ (The grid is on the next page.)

	A quadratic equation can have one real solution,  two real solutions,  or no real solutions. 

	In Example 1, we examined the case where the quadratic equation has two real solutions. Now let's look at a quadratic equation that has one real solution.
***Example 2*:**	Find the solutions of the following quadratic equation by graphing: $x^2 - 4x + 4 = 0$ First, graph the quadratic function, $f(x) = x^2 - 4x + 4$ 1. Find the *y*-intercept. The *y*-intercept is the point where the curve crosses the *y*-axis: (0, *c*). *y*-intercept: (0, 4). 2. Find the vertex. The axis of symmetry: $x = -\frac{b}{2a} = 2.$ *y*-value of the vertex: $y = f(2) = 0$ vertex: (2, 0). 3. Find the reflection of the *y*-intercept about the axis of symmetry. reflection: (4, 4). 4. Graph these three points. 5. Draw a smooth curve that touches the *x*-axis.

	 The only root appears to be, $(2,0)$. Let's substitute it into the quadratic equation. $x^2 - 4x + 4 = 0$ $x = 2 \Rightarrow$ $(2)^2 - 4(2) + 4 = 4 - 8 + 4 = 0$ ✓ The solution of the quadratic equation $x^2 - 4x + 4 = 0$ is, $x = 2$.

Exercise 2:

Find the solutions of the following quadratic equation by graphing:

$$x^2 - 2x + 1 = 0$$

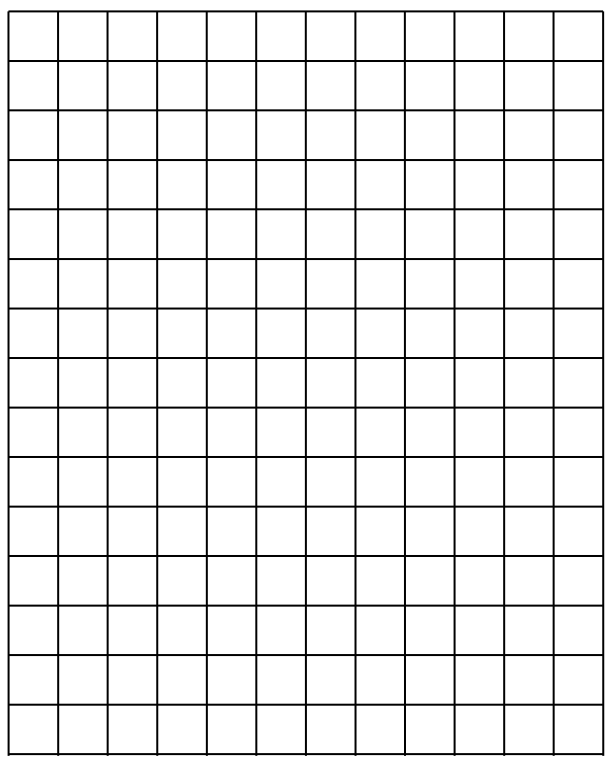
1. Find the y-intercept.

2. Find the vertex.

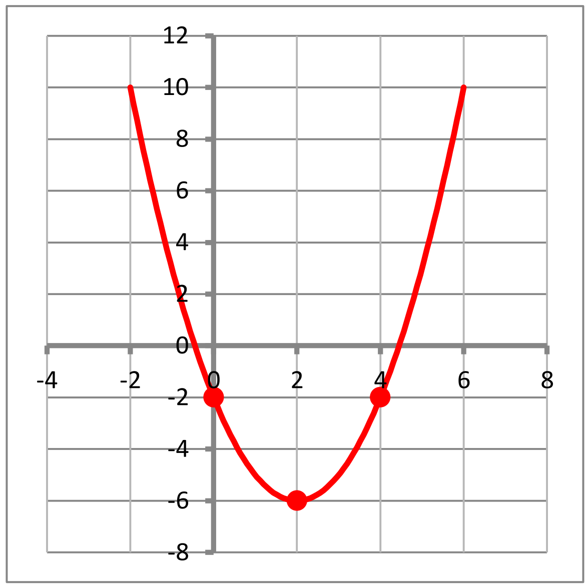
3. Find the reflection of the y-intercept about the axis of symmetry.

4. Graph these three points.

5. Draw a smooth curve that touches the x-axis.

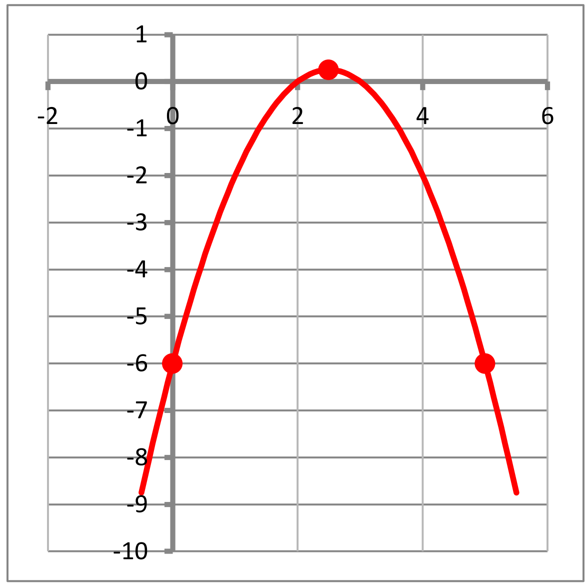


	Estimate the solution and check it with the original quadratic equation.
***Exercise 2 continued*:**	Find the solutions of the following quadratic equation by graphing: $10 - x^2 = 35 - 10x$ (The grid is on the next page.)

	The examples and exercises we have looked at so far, were carefully chosen to have integral solutions. However, you cannot count on this. In fact, most of the time, the roots of the quadratic function will not be integers. In fact, they may not even be rational.
***Example 3*:**	The quadratic function, $y = x^2 - 4x - 2$ is graphed below. Clearly, the graph of the function does not intersect the *x*-axis at integral values.  The solutions of $x^2 - 4x - 2 = 0$ are not integers. An estimate appears to be, $x = -0.5$ and $x = 4.5$ If we substitute these values into the quadratic function, we get, $f(-0.5) = (-0.5)^2 - 4(-0.5) - 2$ $= 0.25 + 2 - 2 = 0.25 \neq 0$ and $f(4.5) = (4.5)^2 - 4(4.5) - 2$ $= 20.25 - 18 - 2 = 0.25 \neq 0$

	Graphing cannot always give us an exact answer; nevertheless, we can find an approximate answer. Nevertheless, we can say that one solution lies between -1 and 0, and the other lies between 4 and 5.
Exercise 3**:**	Use graphing to find the approximate solutions of the following quadratic equation: $x^2 + 4x + 2 = 0$

Example 4: Using Quadratic Equations to Solve Problems	Can we find two real numbers whose sum is 5 and whose product is 6? Let x be one of the numbers. Then $5 - x$ is the other. If their product is 6, then $x(5 - x) = 6$ If we distribute the x and subtract 6 from both sides, then we have the quadratic equation, $-x^2 + 5x - 6 = 0$ Let's solve this equation by graphing the quadratic function $f(x) = -x^2 + 5x - 6.$ 1. Find the *y*-intercept. The *y*-intercept is the point where the curve crosses the *y*-axis: (0, *c*). *y*-intercept: (0, −6). 2. Find the vertex. The axis of symmetry: $x = -\frac{b}{2a} = -\frac{5}{2\cdot(-1)} = \frac{5}{2}.$ *y*-value of the vertex: $y = f\left(\frac{5}{2}\right) = \frac{1}{4}$ vertex: $\left(\frac{5}{2}, \frac{1}{4}\right).$ 3. Find the reflection of the *y*-intercept about the axis of symmetry. reflection: $(5, -6).$ 4. Graph these three points. 5. Draw a smooth curve these points.



The solutions appear to be $x = 2$ and $x = 3$.

A quick check shows that they satisfy the conditions:

$$2 + 3 = 5 \qquad \text{and} \qquad 2 \cdot 3 = 6.$$

***Exercise 4*:**

Find two real numbers whose sum is 6 and whose product is 8. Solve this problem by graphing the quadratic function as we did above.

***Example 5*:**

Find two real numbers whose sum is 6 and whose product is 10.

We'll proceed as above.

Let x be one of the numbers. Then $6 - x$ is the other. If their product is 6, then

$$x(6 - x) = 10$$

If we distribute the x and subtract 6 from both sides, then we have the quadratic equation,

$$-x^2 + 6x - 10 = 0$$

Let's solve this equation by graphing the quadratic function

$$f(x) = -x^2 + 6x - 10.$$

1. Find the y-intercept.

   The y-intercept is the point where the curve crosses the y-axis: $(0, c)$.

   y-intercept: $(0, -10)$.

2. Find the vertex.

   The axis of symmetry: $x = -\frac{b}{2a} = -\frac{6}{2\cdot(-1)} = 3.$

   y-value of the vertex: $y = f(3) = -1$

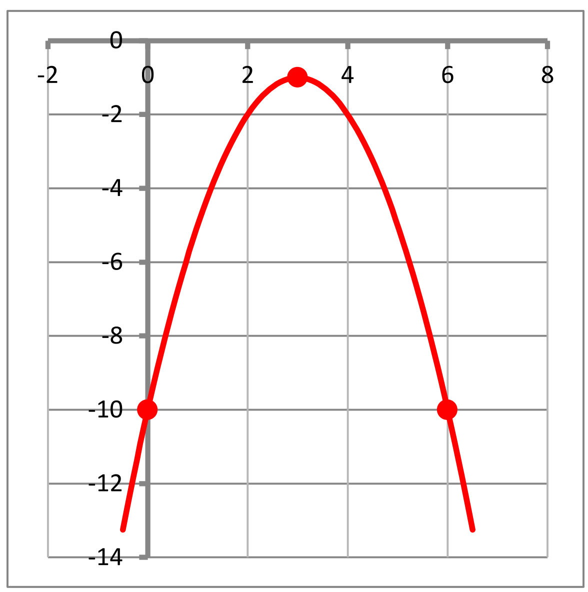
   vertex: $(3, -1).$

3. Find the reflection of the y-intercept about the axis of symmetry.

   reflection: $(6, -10).$

4. Graph these three points.

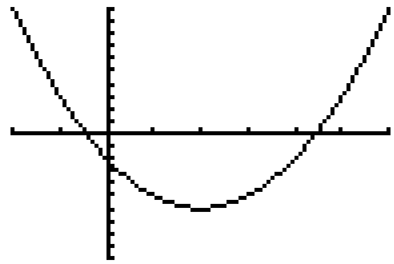
5. Draw a smooth curve these points.



	The graph of $f(x) = -x^2 + 6x - 10$ does not cross the x–axis. This quadratic function has no real zeros. Therefore, the quadratic equation $-x^2 + 6x - 10 = 0$ has no real roots, and there are no two real numbers whose sum is 6 and whose product is 10. Many quadratic equations have no real roots. One of them is $-x^2 + 6x - 10 = 0$.
Exercise 5**:**	Find two real numbers whose sum is 4 and whose product is 8.

***Using a Graphing Calculator*:**

In Example 3, we found that we could only approximate the solutions of $x^2 - 4x - 2 = 0$. Using a graphing calculator, we can find more precise values. The screen capture below shows a graph of $y = x^2 - 4x - 2$. Each tic mark is one unit.

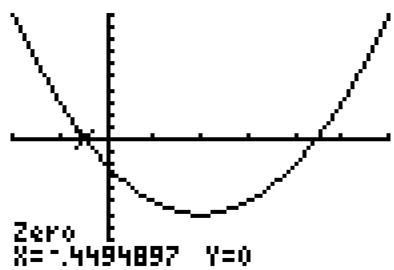


We can use the **TABLE** function on the calculator to find a better approximation:

X	Y1
-.4498	.00152
-.4497	.00103
-.4496	5.4E-4
-.4495	5E-5
-.4494	-4E-4
-.4493	-9E-4
-.4492	-.0014

X=-.4495

We can also use the **CALC** function to find the zeros:



Class work:	p 233: 1-13
Homework:	p 233: 14-19, 21-29 odd, 30-32, 33-51 odd