Algebra II

Name ______________________________

Period ________

Date: Unit **2: Quadratic Functions** Lesson **1: Graphing Quadratic Functions**	Essential Question: **What part of a quadratic function determines whether the function will have a minimum or a maximum? Explain why this is so.**
Standard: A-SSE.1.a	Interpret parts of an expression, such as terms, factors, and coefficients.
***Learning Target*:**	Graph quadratic functions. Find and interpret the maximum and minimum values of a quadratic function. 80% of the students will be able to determine the maximum or minimum of the following function and state its coordinates: $$f(x) = 3x^2 - 2x - 4$$
***Quadratic Function*:**	A ***quadratic function*** is any function in which the greatest exponent is 2. In general, a quadratic function can be written in the following form: $$f(x) = ax^2 + bx + c,$$ where $a \neq 0$. The factors a, b, and c are, typically, constants, and they are called ***coefficients***. a: quadratic or leading coefficient. b: linear coefficient. c: constant or constant coefficient.
Summary	

Define the following terms:	
Function	
Quadratic	
Coefficient	
Leading coefficient	
Linear coefficient	
Constant coefficient	
Maximum	
Minimum	

Parabola:	The graph of a quadratic function is a parabola. A parabola is a U-shaped figure, and it can open either up or down.
	Define the following term:
Parabola	

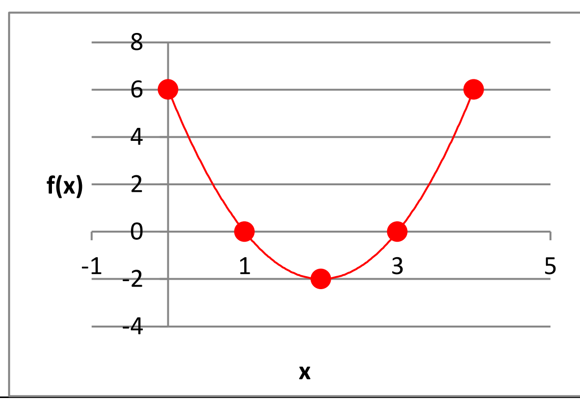
***Example 1*:**

Graph the following quadratic function by making a table of values:

$$f(x) = 2x^2 - 8x + 6$$

1. Choose integer values for x, and enter them into a table.
2. Evaluate the function for each value, and enter the values into the table.
3. Graph the resulting coordinate pairs.
4. Draw a smooth curve through the points.

x	$f(x) = 2x^2 - 8x + 6$	$f(x)$	$(x, f(x))$
0	$f(0) = 2 \cdot 0^2 - 8 \cdot 0 + 6$	6	$(0,6)$
1	$f(1) = 2 \cdot 1^2 - 8 \cdot 1 + 6$	0	$(1,0)$
2	$f(2) = 2 \cdot 2^2 - 8 \cdot 2 + 6$	-2	$(2,-2)$
3	$f(3) = 2 \cdot 3^2 - 8 \cdot 3 + 6$	0	$(3,0)$
4	$f(4) = 2 \cdot 4^2 - 8 \cdot 4 + 6$	6	$(4,6)$



Exercise 1:

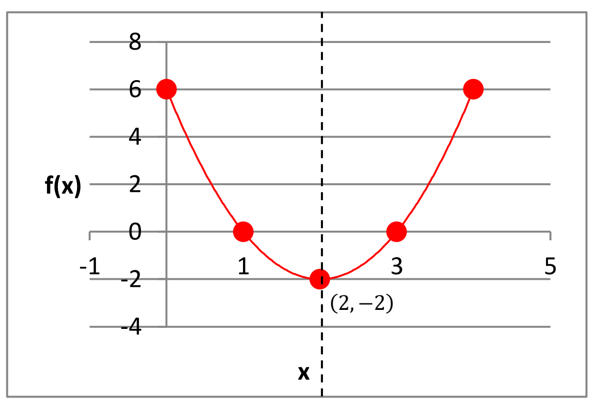
Graph each function by making a table of values.

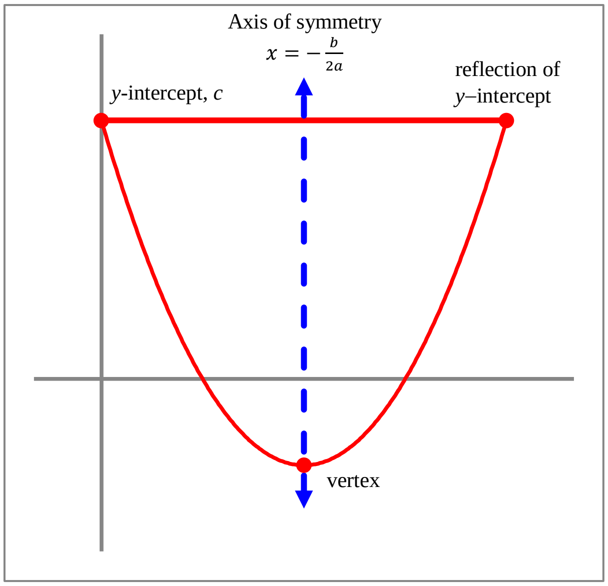
1. $f(x) = -2x^2 + 8x - 3$

x	$f(x) = -2x^2 + 8x - 3$	$f(x)$	$(x, f(x))$
0			
1			
2			
3			
4			

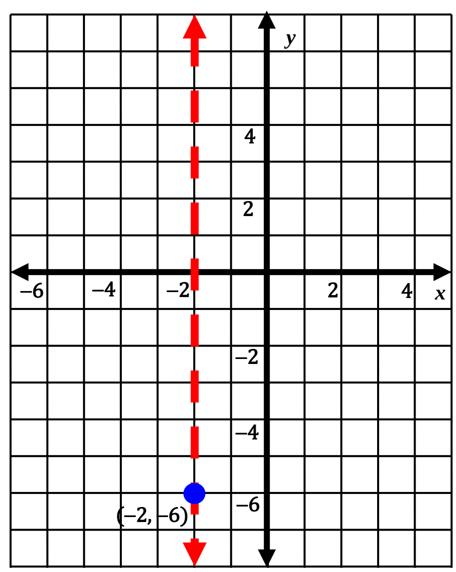
2. $f(x) = 4x^2 - 8x + 1$

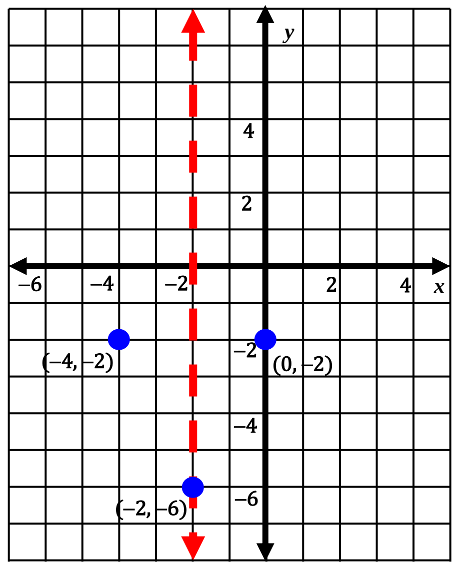
x	$f(x) = 4x^2 - 8x + 1$	$f(x)$	$\left(x, f(x)\right)$
-1			
0			
1			
2			
3			

***Axis of Symmetry*:**	You will notice in Example 1 and Exercise 1 that the graphs display a pattern. They are symmetric. That is, the left half looks like the right half reflected in a mirror. The vertical line on which this mirror would be placed is called the ***axis of symmetry***. In Example 1, the axis of symmetry is the vertical line, $x = 2.$ What are the equations for the axes of symmetry for the two functions in Exercise 1? 1. 2.
***Vertex*:**	The axis of symmetry of a quadratic function intersects the function at one and only one point. This point is called the ***vertex***. The vertex in Example 1 is $(2, -2)$. The figure below is the same as the figure for Example 1 with the axis of symmetry (dashed line) and the coordinates of the vertex added.  Notice that the both first and last points have the same *y*-coordinate $(y = 6)$ and are two units away from the *x*-coordinate of the vertex. Moreover, both the second and fourth points have the same *y*-coordinate $(y = 0)$ and are one unit away from the *x*-coordinate of the vertex. The graph is symmetric about the line, $x = 2$.

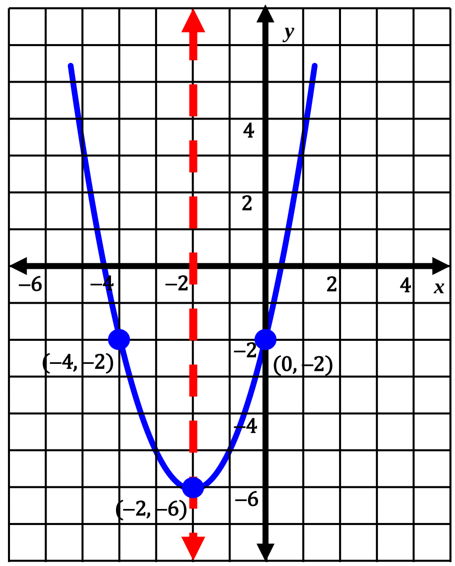
***y–intercept*:**	The graph of the parabola will intercept the *y*–axis when *x* = 0. Therefore, the *y*–intercept is, $$y = a(0)^2 + b(0) + c$$ $$y = c$$
***Axis of Symmetry*:**	The equation of the axis of symmetry is, $$x = -\frac{b}{2a}$$
***Reflection of y–intercept*:**	The reflection of the *y*–intercept has coordinates, $$\left(-\frac{b}{a}, c\right)$$ The figure below illustrates these features. 

Define the following terms.	
Axis of Symmetry	
Vertex	
y–intercept	
Reflection about axis of symmetry	

	You may use the axis of symmetry to graph a parabola. Consider, $$y = x^2 + 4x - 2$$ 1. The axis of symmetry is, $$x = -\frac{b}{2a} = -\frac{4}{2} = -2$$ The y–coordinate of the vertex is, $$y = (-2)^2 + 4(-2) - 2 = -6$$ The coordinates of the vertex are, $(-2, -6)$ Draw a grid and graph the axis of symmetry (dashed, red line) and vertex (blue dot), 

	2. Find the *y*-intercept and its reflection. $y = c = -2$ $(0, -2)$ and $(-4, -2)$ Graph the *y*-intercept and its reflection (blue dots), 

	3. Draw a smooth curve through the points.



	Let's use this approach to graph a quadratic equation.
***Example 2*:**	Graph $f(x) = x^2 + 4x - 3$.

1. Find the *x*-coordinate of the vertex.

   a. Identify the coefficients

   $a = 1$

   $b = 4$

   $c = -3$

   b. Find the *x*-coordinate of the axis of symmetry.

   $$x = -\frac{b}{2a} = -\frac{4}{2 \cdot 1} = -2$$

2. Find the *x*-coordinates of the *y*-intercept and its symmetric point.

a. The *x*-coordinate of the *y*-intercept is, by definition,

$$x = 0$$

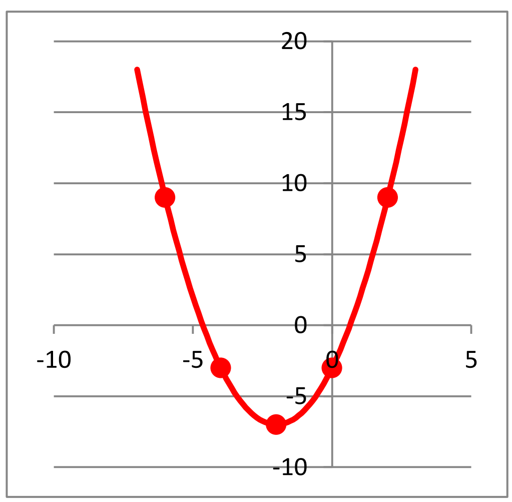
b. The *x*-coordinate of the symmetric point is two times the *x*-coordinate of the vertex.

$$x = 2\left(-\frac{b}{2a}\right) = 2\left(-\frac{4}{2 \cdot 1}\right) = -4$$

3. Make a table of values that includes these three *x*-values. Also include a fourth *x*-value and its symmetric value. Then complete the table.

x	$f(x) = x^2 + 4x - 3$	$f(x)$	$(x, f(x))$
-6	$f(-6) = (-6)^2 + 4(-6) - 3$	9	$(-6,9)$
-4	$f(-4) = (-4)^2 + 4(-4) - 3$	-3	$(-4,-3)$
-2	$f(-2) = (-2)^2 + 4(-2) - 3$	-7	$(-2,-7)$
0	$f(0) = (0)^2 + 4(0) - 3$	-3	$(0,-3)$
2	$f(2) = (2)^2 + 4(2) - 3$	9	$(2,9)$

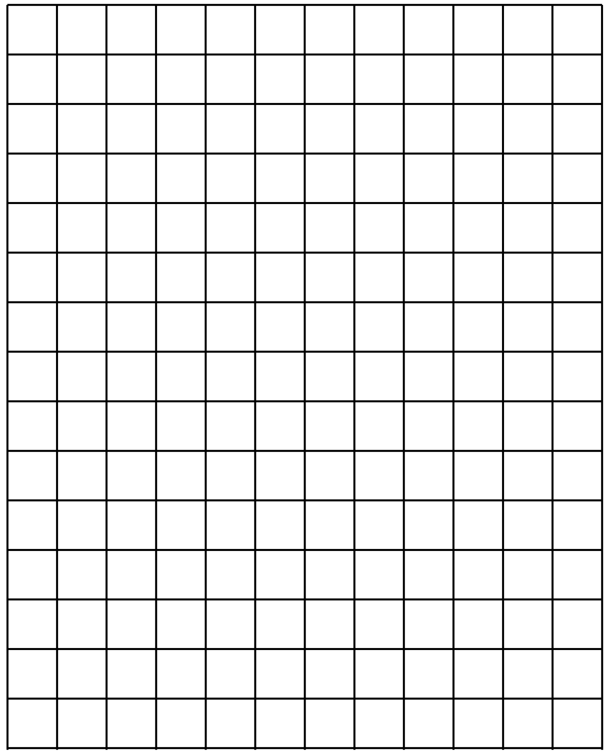
4. Graph the points and draw a smooth curve through them.



Exercise 2:	Graph $f(x) = -5x^2 - 10x + 6$ 1. Find the *x*-coordinate of the vertex. a. Identify the coefficients $a =$ $b =$ $c =$ b. Find the *x*-coordinate of the axis of symmetry. $x = -\frac{b}{2a} = -\frac{}{} =$ 2. Find the *x*-coordinates of the *y*-intercept and its symmetric point. c. The *x*-coordinate of the *y*-intercept is, by definition, $x = 0$ d. The *x*-coordinate of the symmetric point is two times the *x*-coordinate of the vertex. $x = 2\left(-\frac{b}{2a}\right) =$ 3. Make a table of values that includes these three *x*-values. Also include a fourth *x*-value and its symmetric value. Then complete the table.

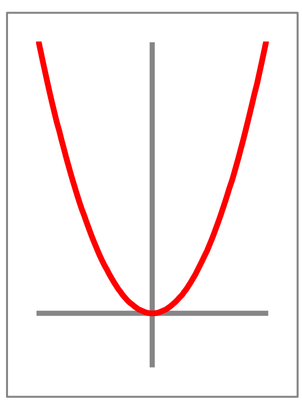
x	$f(x) = -5x^2 - 10x + 6$	$f(x)$	$(x, f(x))$

4. Draw the coordinate axes. Then graph the points from the table. Finally, draw a smooth curve through the points.

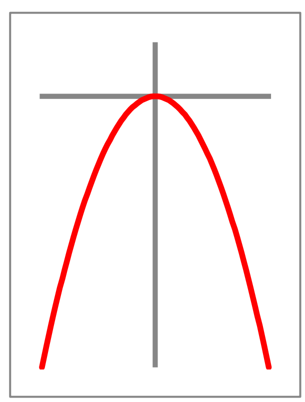


***Extremum*:**

An extremum is an extreme point of a function. It is either a ***maximum*** or a ***minimum*** value. The following figures depict a parabola with a minimum and one with a maximum.



$f(x) = x^2$



$f(x) = -x^2$

a > 0	The figure on the left has a minimum, and $a = 1$. If $a > 0$, then the parabola opens up, and it has a minimum
a < 0	The figure on the right has a maximum, and $a = -1$. If $a < 0$, then the parabola opens down, and it has a maximum
Define the following terms.	
Extremum	
Maximum	
Minimum	
***Example 3*:**	Determine the minimum or maximum value of the following function and state its domain and range: $$f(x) = -4x^2 + 12x + 18$$ 1. Determine whether the function has a ***maximum*** or a ***minimum*** value. $a = -4$. Therefore, the graph opens down, and the graph has a maximum value. 2. Find the maximum or minimum value of the function. The maximum value of the function is the *y*-coordinate of the vertex.

The *x*-coordinate of the vertex is,

$$x = -\frac{b}{2a} = -\frac{12}{2(-4)} = \frac{3}{2}$$

Find the *y*-coordinate of the function by evaluating the function for $= \frac{3}{2}$.

$$f\left(\frac{3}{2}\right) = -4\left(\frac{3}{2}\right)^2 + 12\left(\frac{3}{2}\right) + 18$$

$$f\left(\frac{3}{2}\right) = -9 + 18 + 18 = 27$$

The coordinates of the vertex are, $(1.5, 27)$.

3. State the domain and range of the function.

The domain of any quadratic function of the form, $f(x) = ax^2 + bx + c$ where a, b, and c are real numbers is the set of real numbers.

Since this function has a maximum value, the range is all real numbers less than or equal to the *y*-coordinate of the vertex,

$$\{f(x) | f(x) \leq 27\}$$

***Exercise 3*:**	Using Example 3 as a pattern, determine the minimum or maximum value of the following function and state its domain and range: $$f(x) = 4x^2 - 24x + 11$$
Class work:	p 224: 1-11
Homework:	p 224: 13-39 odd, 42, 58-62, 64