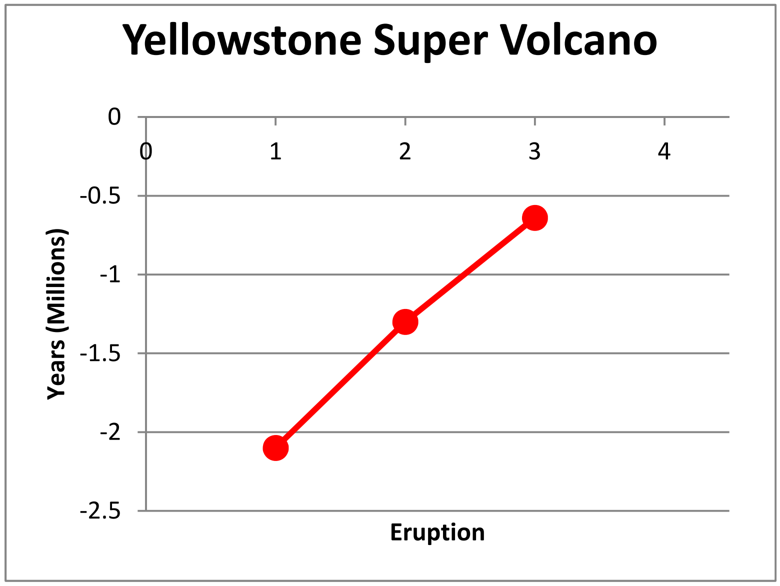
Name ______________________________________

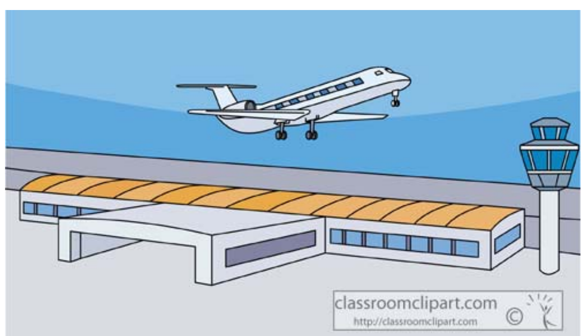
Period ___________

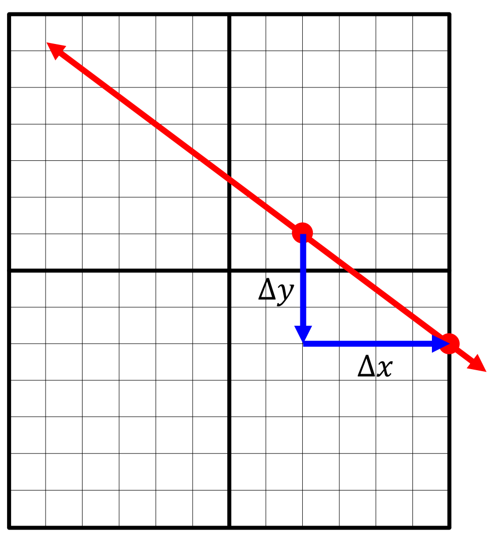
Date: Unit: **1 Linear Equations and Functions** Lesson: **2 Rate of change and Slope**	Essential Question: **Why doesn't a vertical line have a slope?**
Standard: F-IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
***Learning Target*:**	To find the slope of a line and to graph a line given its slope and a point on it. 80% of the students will be able to find the slope of the line that passes through the points (1, –3), (–2, –4).  Yellowstone National Park is renowned for its geothermal features such as geysers and hot springs. These features result from the fact that Yellowstone National Park lies on top of a super volcano, the Yellowstone Super Volcano. This super volcano is known to have erupted three times during the last several million years. The last eruption occurred 640,000 years ago. The graph on the next page shows the dates of the previous three eruptions.
Summary	



What is the average rate of change of this graph? If this trend continues, when would you expect the Yellowstone Super Volcano to next erupt?

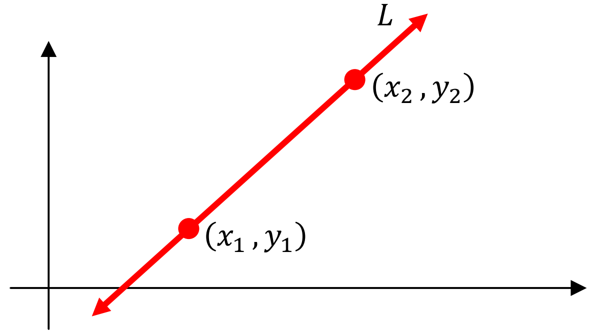
At Montgomery Field in San Diego, the noise created by a departing jet cannot exceed 70 decibels on the ground in adjacent residential areas at night. A jet plane must take off at an angle so that it will be high enough over the residential areas to ensure the noise on the ground does not exceed this value.



Slope:	Suppose the jet must be Δx feet above the ground to meet this requirement. If the residential area is Δy feet from the take off point, then the path of the plane shown is a line that rises Δx feet for each Δy feet traveled horizontally. The steepness, or ***slope***, of the path is, $$\textbf{slope} = \frac{\textbf{rise}}{\textbf{run}} = \frac{\Delta y}{\Delta x}$$
Slope of a line:	The slope, *m*, of a line is the change in *y* (Δ*y*) divided by the change in *x* (Δ*x*). $$m = \frac{\Delta y}{\Delta x}$$ 

You can use this same method to measure the steepness, or ***slope***, of a non-vertical line L in a coordinate plane. First, choose any two distinct points (x_1, y_1) and (x_2, y_2) on L. (See the figure below. You read (x_1, y_1) as "x one, y one" or "x sub one, y sub one".) Then,

L



$$\text{slope of line } L = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \quad (x_2 \neq x_1)$$

***Example 1*:**

Find the slope of the line that passes through the points

a. $(3, 2)$ and $(5, 6)$

$\Delta y = y_2 - y_1 = 6 - 2 = 4$

$\Delta x = x_2 - x_1 = 5 - 3 = 2$

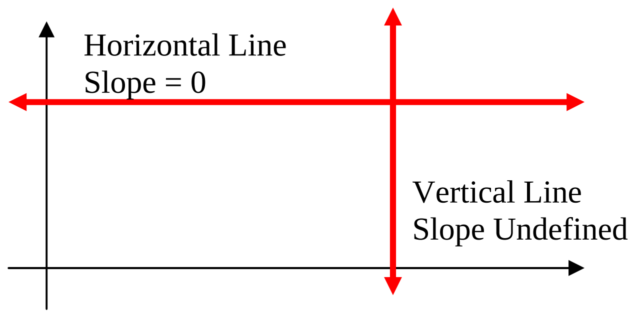
$m = \frac{\Delta y}{\Delta x} = \frac{4}{2} = 2$

b. $(3, -2)$ and $(-5, 6)$

$\Delta y = y_2 - y_1 = 6 - (-2) = 8$

$\Delta x = x_2 - x_1 = (-5) - 3 = -8$

$m = \frac{\Delta y}{\Delta x} = \frac{8}{-8} = -1$

Exercise 1:	Find the slope of the line containing the points, (4, 3) and (0, 1). Find the slope of the line containing the points, (6, –5) and (3, 5).
Vertical and Horizontal Lines:	Consider the figure below. If line L is vertical, then x_2 equals x_1. Therefore, $x_2 - x_1 = 0$, and $\frac{y_2 - y_1}{x_2 - x_1}$ does not represent a real number. (You cannot divide by zero.)  ***Vertical lines have no slope!*** If L is horizontal, then y_2 equals y_1. Therefore, $y_2 - y_1 = 0$, and $\frac{y_2 - y_1}{x_2 - x_1} = 0$. ***Horizontal lines have slope 0.*** Find the slope of the line containing the points, (–2, 1) and (4, –1).

$$slope = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-1) - 3}{4 - (-2)} = \frac{-4}{6} = -\frac{2}{3}$$

Find the slope of the line containing the points, (3, –4) and (3, 5).

$$slope = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-4)}{3 - 3} = \frac{9}{0}$$

The line is vertical and has no slope.

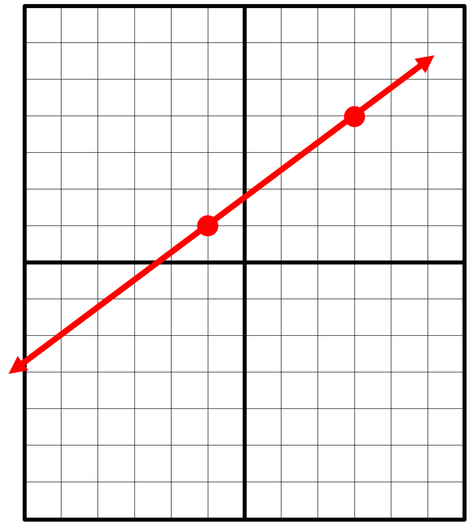
***Finding the Slope*:**

When you the graph of a line, you can find the slope by identifying two points, and dividing the change in *y* by the change in *x*.

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

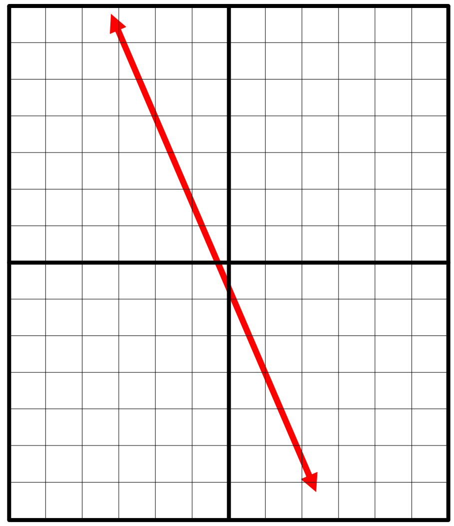
***Example 2*:**

In the graph below, the points (–1, 1) and (3, 4) lie on the line.

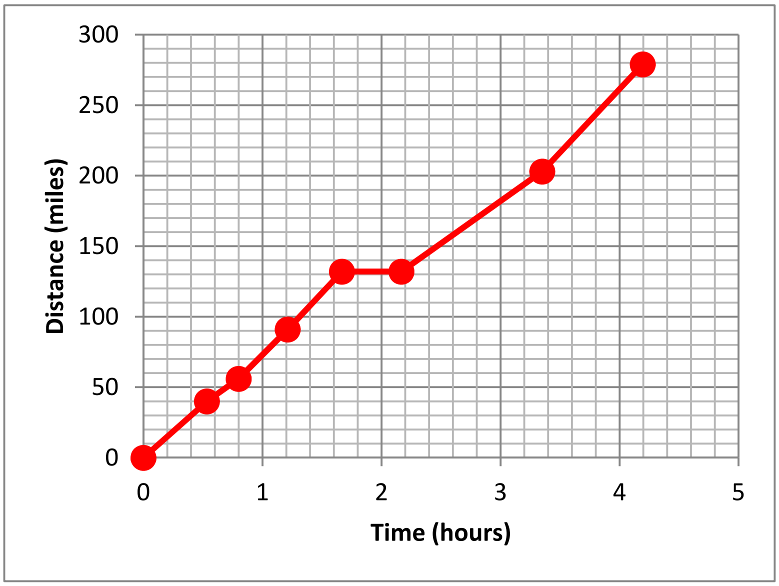


Therefore, the slope is,

$$m = \frac{\Delta y}{\Delta x} = \frac{4 - 1}{3 - (-1)} = \frac{3}{4} = 0.75$$

***Exercise 2*:**	Find the slope of the line in the following graph: 
***Rate of Change*:**	***Rate of change*** is the ratio of the change in one variable compared to the change in another variable. It is closely related to the slope. In the strictest sense, the slope is the instantaneous rate of change. Rate of change is often used for the ***average change*** over an interval. It is also used to describe the change of some quantity over a time interval.

Example 3:	Suppose a crane can unload 12 containers from a container ship in 1 hour. Then the rate of change is $$rate\ of\ change = \frac{change\ in\ y}{change\ in\ x}$$ $$= \frac{change\ in\ containers}{change\ in\ time} = \frac{12}{1} = 12$$ 
Exercise 3:	Suppose a pump is filling a pool. After 9:00 AM, the pool has 96 gallons in it. After 6:00 PM, the pool has 528 gallons in it. What is the rate at which the pump fills the pool?

***Example 4*:**	Alexandria left Santa Fe and drove to Las Cruces. She stopped in Socorro for half an hour to eat lunch. This is a graph of her distance *vs*. time. The dots represent cities she passed along the way. What was her average speed?  She arrived in Las Cruces after traveling about 277 miles in 4.2 hours. Therefore, her average speed is, $speed = \frac{distance}{time} = \frac{277-0}{4.2-0} = 66.0\ mi/h$
***Exercise 4*:**	Solve the Yellowstone super volcano problem.

Class work: p 79: 1-8

Homework: p 79: 9-21 odd, 22, 23-27 odd, 30, 31-35 odd, 36, 45-57