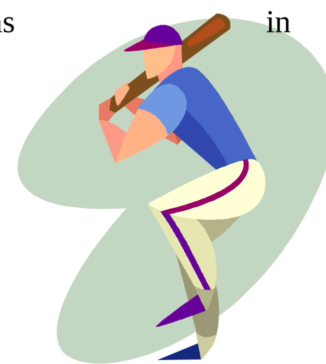
Algebra II

Name ________________________________

Period __________

Date: Unit **1: Linear Equations and Functions** Lesson **1: Solving Equations**	Essential Question: **What is the difference between the phrases, "three less than a number" and "three decreased by a number"?**
Standard: A-SSE.1	Interpret expressions that represent a quantity in terms of its context.
Learning Target:	80% of the students will be able to solve the formula $h = -16t^2 + vt$ for v. A baseball player has hit 26 home runs in the first 130 games of the season. He can expect to play 30 more games during the rest of the season. If he continues to hit home runs at the same average rate, how many home runs will he hit for the entire season? 

Summary

If the player wants to hit 35 home runs for the season, what is the average number of home runs he must hit each day for the remainder of the season?

To use algebra as a problem-solving tool, you often must translate word phrases into algebraic expressions.

***Example 1*:**

Represent each word phrase by an algebraic expression. Use n for the variable.

A number decreased by 2	$n - 2$
Five more than three times a number	$3n + 5$
The difference between a number and its square	$n - n^2$
The sum of twice a number and 6	$2n + 6$
Twice the sum of a number and 6	$2(n + 6)$

The answer to the third expression in Example 1 is $n - n^2$ and ***not*** $n^2 - n$. In this course, when we say "the difference between *x* and *y*," we mean $x - y$.
Similarly, "the quotient of *x* and *y*" means $\frac{x}{y}$ or $\div y$.

***Exercise 1*:**	Represent each word phrase by an algebraic expression. Use *n* for the variable. A number increased by 2 Six less than four times a number The difference between a the square of a number and twice the number The sum of half a number and 3 Twice the sum of a number and half the number's square
***Example 2*:**	Ann is biking at *r* mi/h. Use the variable *r* to represent each word phrase by an algebraic expression. Ann's speed if she bikes 5 mi/h slower $r - 5$ Ann's speed if she bikes 3 mi/h faster $r + 3$ The average of Ann's and Juan's speeds if Juan bikes at 10 mi/h $\frac{r + 10}{2}$

***Exercise 2*:**	A ship is sailing at *r* knots. Use the variable *r* to represent each word phrase by an algebraic expression. Another ship's speed if the first ship is overtaking it at 6 knots. An oncoming ship's speed if the two ships are closing at 30 knots. The ship's speed if the captain increases its speed by 5 knots. The ship's speed if the captain reduces its speed by one half. The ships speed if the captain increases its speed by one half. The ship's speed after it gets stuck in a sand bar.

Mathematical Sentence:	"A statement that shows a relationship between either numeric or algebraic expressions. Equations and inequalities are two types of mathematical sentences."[1] The following mathematical sentences are true: $23 - 15 = 8$ $\frac{80 - 25}{5} > 10$ The following mathematical sentences are false: $13 - 15 = 2$ $\frac{80 + 25}{5} < 10$
***Open Sentence*:**	An open sentence is a mathematical sentence that contains one or more variables. The following open mathematical sentences can be true or false, depending on the values of the variables: $23x - 15 = 8$ $\frac{80x - 25y}{5} > 10$
***Equation*:**	An ***equation*** is a mathematical sentence in which two expressions are equal to each other. $23 - 15 = 8$ $2x - 15 = 5$

[1] http://www.mathresources.com/products/insidemath/maa/mathematical_sentence.html

***Translating Words into Symbols*:**

Word	Symbol
sum	$+$
and	
more	
plus	
difference	$-$
less	
minus	
times	$\times \quad \cdot$
product	
ratio	$\div \quad \frac{p}{q}$
quotient	
per	
divide	
is	$=$
equals	

***Example 3*:**

Write a verbal sentence to replace each equation.

$$\frac{x}{5} = 3$$

The quotient of a number and 5 is 3.

$$2 + n = -2$$

The sum of 2 and a number is −2.

$$n - 3 = 15$$

3 less than a number is 15.

***Exercise 3*:**	Write a verbal sentence to replace each equation. a. $f - 6 = 8$ b. $3y = y^3 - 5$
Properties of Equality:	$\forall\, \boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c} \in \mathbb{R}$
Reflexive	$a = a$
Symmetric	$a = b \Rightarrow b = a$
Transitive	$a = b \; and \; b = c \Rightarrow a = c$
Addition	$a = b \Rightarrow a + c = b + c$ and $c + a = c + b$
Multiplication	$a = b \Rightarrow ac = bc$ and $ca = cb$

***Example 4*:**	Name the property of equality illustrated in each statement. 1. $x + 4 = 3 \Rightarrow 2(x + 4) = 6$ **Multiplication Property** 2. $z = a + 2$ and $a + 2 = 3 \Rightarrow z = 3$ **Transitive Property** 3. $x = y + z \Rightarrow y + z = x$ **Symmetric Property**
***Exercise 4*:**	Name the property of equality illustrated in each statement. 1. $14 + x = 25 \Rightarrow 25 = 14 + x$ 2. $x = y \Rightarrow 3x = 3y$ 3. $y = z \Rightarrow y + 5 = z + 5$ 4. $x = y \; and \; y = 21 \Rightarrow x = 21$ 5. $22 + w = b \Rightarrow b = 22 + w$ 6. $r + 17 = s \Rightarrow r = s - 17$

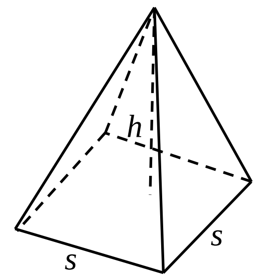
Example 5*: *One Step	Solve each equation. Check your solution. a. $v + 5 = 4$ $v + 5 - 5 = 4 - 5$ $v = -1$ $(-1) + 5 = 4$ ✓ b. $a - 13 = 33$ $a - 13 + 13 = 33 + 13$ $a = 46$ $46 - 13 = 33$ ✓ c. $4y = 16$ $\frac{4y}{4} = \frac{16}{4}$ $y = 4$ $4 \cdot 4 = 16$ ✓ d. $-\frac{3}{4}x = 27$ $\left(-\frac{4}{3}\right) \cdot \left(-\frac{3}{4}\right)x = \left(-\frac{4}{3}\right) \cdot 27$ $x = -36$ $-\frac{3}{4}(-36) = 27$ ✓

Exercise 5:	Solve each equation. Check your solution. a. $x + 2.54 = 16.32$ b. $y - 2 = 3.6$ c. $2z = 38$ d. $\frac{a}{15} = -2$

Example 6*:** ***Multi- Step	To solve an equation requiring more than one operation, undo each operation in turn. Solve $5(x+3)+2(1-x)=14$ $5x+5\cdot3+2\cdot1-2x=14$ distributive property $5x+15+2-2x=14$ simplify $5x+17-2x=14$ simplify $5x-2x+17=14$ commutative prop $(5-2)x+17=14$ distributive prop $3x+17=14$ simplify $3x=-3$ additive prop $x=-1$ multiplication prop Check $5\big((\mathbf{-1})+3\big)+2\big(1-(\mathbf{-1})\big)=14$ $5\cdot2+2\cdot2=14$ $14=14$ ✓

***Exercise 6*:**	Solve each equation. a. $-10x + 3(4x - 2) = 6$ b. $2(2x - 1) - 4(3x + 1) = 2$

***Formula*:**	A ***formula*** is an equation that states a relationship between two or more variables. The variables usually represent physical or geometric quantities. For example, the formula $$h = -16t^2 + vt$$ gives the height *h* (in feet) of a launched object *t* seconds after firing with initial velocity *v* (in ft/s). Given values for all but one of the variables in a formula, you can find the value of the remaining variable.
***Example 7*:**	A model rocket launched with initial velocity *v* reaches a height of 40 ft after 2.5 s. Find *v*. Begin with the formula for height. Then solve for *v*. $h = -16t^2 + vt$ $h + \mathbf{16t^2} = -16t^2 + \mathbf{16t^2} + vt = vt$ $\frac{h + 16t^2}{\mathbf{t}} = \frac{vt}{\mathbf{t}} = v$ $v = \frac{h + 16t^2}{t} = \frac{40 + 16(2.5)^2}{2.5} = 56\ ft/s$ ***When you solve a formula or equation for a certain variable, you can think of all the other variables as constants, that is, as fixed numbers. Then solve by the usual methods.***

Exercise 7:	The volume V of a pyramid with height h and sides s is given by the formula, $V = \frac{1}{3}s^2h$ Solve this formula for h. 

	There are often different ways to solve a problem. Using the properties of equality can help you find a simpler way.
***Example 8*:**	If $6x - 12 = 18$, what is the value of $6x + 5$?
Standardized Test	A 5 B. 11 C. 35 D. 41
	Since you are asked to find the value of $6x + 5$, you do not need to find the value of x. Use the Addition Property of Equality to make the left side of the equation $6x + 5$.
	$6x - 12 + \mathbf{17} = 18 + \mathbf{17}$
	$6x + 5 = 35$
	The answer is C.
***Exercise 8*:**	If $5y + 2 = \frac{8}{3}$, what is the value of $5y - 6$?
	A $-\frac{20}{3}$ B. $-\frac{16}{3}$ C. $\frac{16}{3}$ D. $\frac{32}{3}$
Class work:	p 22: 1-21
Homework:	p 22: 23-51 odd, 52, 53-57 odd, 59-62, 64-82